INNOVATION AND EVOLUTION IN AN ARTIFICIAL SPATIAL ECONOMY

BASTIAAN STRAATMAN

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by

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Invention, strictly speaking, is little more than a new combination of those images which have been previously gathered and deposited in memory: nothing can come of nothing. (Sir Joshua Reynolds, Discourse II, 1769, lines 83-87)

Abstract

This thesis regards economies as constructive systems. A constructive system is defined as a system whose later components are generated during the interaction of its earlier components. The thesis develops a model that simulates an evolving spatial economy: as time progresses, it generates new products and new technology. The model is built around von Neumann technology matrices, but this model does not assume fixed matrices as is usual. Instead it progressively expands the matrices by adding columns and rows representing new technologies and products. The additions are not predetermined, but are composed of new combinations of existing products and techniques, and by this means new, innovative functionality is created. A spatial, individual based model, consisting of economic agents and a price mechanism, decides where the profits are, and thus selects the successful additions. The model shows that through the localized price mechanism and agents' local optimization an efficient global economy can exist, illustrative of Adam Smith's invisible hand, and can evolve: according to a measure developed by Bedau *et al.* (1998) the model demonstrates real evolutionary dynamics. New goods and skills allow the economy to enlarge, either through a growing population or through increased consumption. The model allows the study of spatial implications of continuous creativity, which is important in our world of continuously changing economic challenges and opportunities.

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Chapter 1

Introduction

Our economy is one that never stands still. New companies appear and others become obsolete. New products arise, and then disappear again. This constant renewal of elements is one of the most fascinating characteristics of our economy. The diversity of products, and of the mechanisms to manufacture them, is astounding. Despite these characteristics, economics, for the larger part, chooses to ignore this aspect of the economic system (Hodgson, 1993; Blatt, 1983). A descriptive treatment by Schumpeter (1961) and followers is a rare exception. Mainstream economics treats the system as if it were in equilibrium, or at least nearby and moving toward equilibrium. Although the field of economic geography, which in the nineteen sixties closely followed developments in economics, has largely left that quantitative age behind, the descriptive approach that has taken its place does not allow a thorough analysis of the processes of innovation. However, as Dosi *et al.* write, technological change is "now widely recognized to constitute a primary source of industrial dynamics" (Dosi *et al.*, 1997, p.12), or it is even "**the** driving force of economic growth" (Dawid, 2006, p.1237 his emphasis). No wonder the success of these approaches is limited, when the evolving capacities that are responsible for the diversity, and which are recognized as a driving force behind the whole economic system, are completely shut out. This thesis addresses the lack of attention to innovation and diversification of our economy, and proposes a model framework to include evolution in our analysis of the economic environment we live in. A useful theory of economic geography must be based on an appropriate economic framework. Once an appropriate economic theory is developed, one that deals with endogenous change, this newly developed theory can be extended to include spatial considerations, and thus contribute to the field of economic geography.

1.1 The road to dynamic modelling

Currently, economic geography is going through a transition phase (Boschma and Frenken, 2006). One can think of several reasons for this. Firstly, since the quantitative revolution in the nineteen sixties, more and more researchers have realized that neoclassical equilibrium theory, which was borrowed from economics, is incapable of keeping up with reality. Location theory tells us what spatial distribution of services to expect based on a given set of economic activities, assuming that physical suitability, infrastructure, and government, as well as the set of economic activities remain the same. By its very nature, a location theory based on equilibrium economic theory is incapable of explaining phenomena such as change, heterogeneity, persistence of diversity, turbulent dynamics, and at the same time, invariant structural patterns (Dosi *et al.*, 1995; Nelson and Winter, 1982; Rigby and Essletzbichler, 1997). Likewise, a competitive market system is an "essentially dynamic" system, Blatt writes (Blatt, 1983, p.5), and in that case, it can not be understood by focusing on a static economic equilibrium. Krugman argues that neoclassical equilibrium theory chooses to ignore issues such as development economics, because the formal tools regarded as acceptable in the field are inappropriate to deal with these issues (Krugman, 1995, p.68). Most importantly, none of the existing theories are capable of dealing with the question of how and why economies are expanding and becoming more complex, with the diversity of technology and products increasing all the time (Kauffman, 1995, 2000).

Awareness of the limitations of the neoclassical approach is not new. In the nineteenth century, Marshall was already arguing that quantitative changes resulting from a shift in demand or price were important, but that qualitative changes were of much greater importance, and that economics should draw from the life sciences instead of physics and equilibrium theory (Hodgson, 1993).

In short term scenarios, change in economic activity may be approached as if the economic system finds itself in a closed environment. The dynamics observed in the system may be the result of the system moving toward a balanced state where supply and demand are equal, which is obviously a situation to be preferred over a mismatch between supply and demand. In such a closed environment, the equilibrium approach does not ignore technology, but it assumes technology to be given (Rosenberg, 1994). Furthermore, it is assumed that any imbalance can be dealt with quantitatively within the given technology set, i.e. a higher demand drives the price up and as a consequence the supply will go up. If we assume complete markets, i.e. if we assume that for every commodity that will be traded in the future, the demand and supply are known, then Arrow and Debreu have shown that *ceteris paribus* there exists an equilibrium at which markets clear. However, what happens when supply cannot be increased, and the equilibrium is not feasible due to certain old or new constraints? In reality, the economic environment is not closed, and an adaptive system such as our economy will change itself qualitatively as well as quantitatively as a reaction to disequilibrium situations that occur. Uncertainty, due to future innovation for example, which is very prominent in our economic system, limits the applicability of equilibrium theory and thus "the wonderful Arrow-Debreu theory is fundamentally flawed" (Kauffman, 2000, p.215).

This thesis addresses that weakness. Innovation and technological change are regarded as a driving force of economic development (Dawid, 2006; Nelson and Winter, 1982; Schumpeter, 1961). The adaptive capacity of an economic system is truly remarkable (Kauffman, 1995, 2000), though adaptations are not always successful (Diamond, 2005). Due to different levels of scarcity, there is a continuous drive to make processes more efficient, or to substitute current, limited inputs with less scarce inputs. Furthermore, the ever diversifying set of commodities creates an ever larger set of possible new consumables and intermediate products and, at the same time, new technological possibilities.

There have been other attempts to deal with the flaws of the equilibrium approach. Following the quantitative revolution, economic geographers became aware of these shortcomings, and increasingly economic geography turned away from the positivist approach just as the opportunity of dynamic modelling arrived. Clearly, something was missing in the neoclassical approach. To a certain extent, the gap was filled by an institutionalist approach: the specifics of the location, and an awareness of the importance of many different actors, have led to a focus on the institutions that make up the environment in which human and economic action takes place (Boschma and Frenken, 2006). In analyzing complex systems, however, a descriptive approach quickly becomes too complicated. Our mind is simply not capable of understanding the aggregate effect of all possible interactions that take place, and therefore some help is required. Computer based simulation is an appropriate method to assist in the study of complex systems, and our evolving economy is most definitely a complex system.

Due to this complexity, awareness of the necessity of dynamic modelling has increased sharply in recent years. Over the last three decades, dynamic modelling has made its way into the research community of economic geographers. This can, in part, be explained by the increasing accessibility of computational power, but at least equally important is the increasing awareness of certain weaknesses in both the analytic neoclassical and the descriptive approaches. Agent based models and cellular automata models are indeed very well suited to capture dynamic economic processes in space. Cellular automata based urban land use models have been quite successful in simulating changing land use patterns (White, 1998), and agent based models, such as adaptive economic agents in agent based economic computations (ACE) (Dawid, 2006), are excellent examples of successful applications of this type of dynamic model. The models are bottom up, or process based, and the interactions of the agents or the cells are quite capable of capturing the dynamics of such systems. The actors are part of each other's environment, and through the interaction of the actors, the environment becomes dynamic and requires the actors to adapt.

However, though these models are dynamic, they still lack innovative qualities. The examples mentioned above may incorporate new elements through exogenous scenario files, or through pre-defined technology sets which already include the innovative techniques that will be available in the coming time, but this still can not be qualified as evolution. Evolution involves the introduction of new products and new techniques that can influence future products and techniques.

As a historical example of this, consider the following sequence of events in the early industrial revolution of England. The invention of a steam driven water pump by Savery quickly improved conditions for mining coal, since prior to this horses had been used to haul the water out of mines. When, due to a mechanical defect in a Savery pump, the power of atmospheric pressure over vacuum was revealed, the safer and more effective Newcomen water pump was developed. Coal could be mined more cheaply with this new pump, and cheaply-fueled coal furnaces led to better quality metals, such as iron and brass. This in turn led to better tools, and better cannons. Precision drilling was developed to make cannons stronger than iron or brass casting allowed, since drilling led to strong, perfect cylinders in the cannons. James Watt then used such drilled cylinders together with a separate condenser to improve the water pumps. In short order, Watt's invention was not merely used to pump water, but also to supply the rapidly increasing demand for power. Watt's steam engine did not initiate the industrial revolution, but it did play a major role in the unfolding of the event. A major reorganisation of the economic system took place, followed by an equally impressive reorganisation of the urban system. New forms of housing, food, fuel and transportation were required, with entrepreneurs seizing the opportunities to gain profit. A similar large scale reorganisation was initiated by the introduction of the combustion engine. It caused the horse based transportation system, with its saddlery, stables, and smithies, to be replaced by one based on the automobile, and required the construction of gravel roads, gas stations, motels, and garages. Ultimately, it led to a major expansion of both tourism and suburbia (Kauffman, 2000, p.216). These are examples of major innovation clusters, but there are also many small innovations that cumulatively are important. Each innovation plays its role in the web of technological progress, and technologies and products coevolve in unforeseen directions.

Therefore, the impact of innovation and diversification is enormous, and needs to be included in the study of economic systems and, by extension economic geography. A small, fixed list of possible future innovations is insufficient. What is necessary, and what this thesis argues for, is an explicit handling of *innovation* in addition to dynamics. This will make the treatment of evolution explicit.

Innovation is a difficult concept to capture in a model. It involves novelty, and how does one anticipate novelty? To a certain extent, the appearance of new elements is a reaction to actual future situations that we cannot foresee. Thus we can not pre-determine novelty, nor do we want to. We need a model capable of generating novelty — new elements, and new relationships — independent of the modeller. Such models are studied in the field of artificial chemistry, and are called constructive models.

1.2 Constructive dynamic modelling

> In a *constructive dynamical system* new components can appear, which may change the dynamics of the system. This is different from a conventional dynamical system where all components and interactions are given at the outset of the process. (Dittrich *et al.*, 2001, p.235)

Agent based models and cellular automata models are considered conventional dynamical systems and do not exhibit evolutionary behaviour. There are no new entities in these models, and no new qualities. Evolution appears when a model itself constructs new components and new relationships among the components. Dynamic modelling theory has to be expanded to include constructive systems.

In this thesis it is argued that the existing field of evolutionary economics and its geographical applications does not include a constructive system. Many, if not all, of the evolutionary economic models focus on changes in labour and capital coefficients, and the possible adaptations open to firms are selected from a predefined list. This makes the models undoubtedly dynamic, but not evolutionary. At best, the models of evolutionary economics display a partial Lamarckian evolution, a Lamarckian adaptation, where changes in the environment (i.e. the market) force agents to adapt through optimizing their routines. This thesis will go further, allowing agents to be truly innovating, developing new products and technology during a model run. As a result, an increasingly complex economy, with a diversifying set of products and technologies, is generated during a model run.

The ultimate aim of this thesis then is to simulate a spatial economy that is both dynamic and evolutionary. This means developing a model with two fundamental characteristics. First, the model must display adaptive behaviour due to the interaction of its parts. Secondly, at the same time the model has to be a constructive system, which means that new variables and new qualities are introduced to the model by the model itself. This is more than just dynamical modelling.

1.3 Space

Evolutionary economics, and evolution in general, depend on variety among the actors in the system. Without variety there is no inheritance of different traits and no selection of the fitter traits. Without variety, there are very restricted possibilities for combining elements into new features. The persistent variety among firms concerning the use of labour, technology, productivity and profits is one of the phenomena that can not be explained by traditional economics. Traditional economics argues that where differences exist, the system will strive for the unique, most efficient production method and variety will disappear; consequently the system can be dealt with analytically. Equilibrium methodology does not easily accommodate firm specifics (Dosi *et al.*, 1995). However, recent research has shown that there is a wide variety of skills and techniques for a single product, and furthermore, there exists spatial variety in production technology (Essletzbichler and Rigby, 2005; Rigby and Essletzbichler, 2006). Essletzbichler and Rigby argue, therefore, that there is sufficient empirical evidence to support the spatial evolutionary economics approach. Although Krugman's new economic geography approach has shown, as did other studies long before him (Allen and Sanglier, 1979; White, 1977, 1978), that agglomeration can occur even in a homogeneous space, other research (see Werker and Athreye (2004) for an overview) shows the importance of local infrastructure, such as formal and informal institutions, traffic and communication links, education, and research and development facilities. Evolutionary economics and evolutionary economic geography agree with the institutionalist approach that these factors differ from region to region, and that many differences in economic activity between regions can be explained thereby. Furthermore, classical production factors (land, labour, capital) differ from one place to the next. The applied production techniques naturally depend on these. Tacit knowledge is another production factor that shows a strong spatial character. Accumulation of knowledge can lead to agglomeration effects (Werker and Athreye, 2004), and another current topic of interest is spillovers from accumulation of knowledge (Audretsch and Feldman, 1986). Space matters, space generates variety, and, space also generates stability (Hofbauer and Sigmund, 1988). Because such a model framework does not yet exist, this thesis will develop a constructive spatial economic system.

The emerging field of evolutionary economic geography (Boschma and Frenken, 2006) adds dynamic aspects to the descriptive institutionalist approach and spatial aspects to evolutionary economics. In addition to explaining differences between regions, evolutionary economic geography models the process of adaptation. However, despite its name, evolutionary economic geography does not include novelty in its models, for the models are not constructive. This thesis will address this issue directly and thereby go beyond evolutionary economic geography as it currently stands. It will describe the development of a constructive system to model innovation and a diversifying spatial economy.

First it will provide an overview of the current state of the art and a review the science of evolving economic systems. Then chapters 3 and 4 will introduce the basic model and a method to expand the economy qualitatively. Chapter 5 describes a typical model run and Chapter 6 results. Chapter 7 introduces a spatial application, and finally Chapter 8 provides a conclusion.

Chapter 2

Background to the problem

This chapter provides the context to the problem and gives sources of inspiration for the approach to follow. First, it discusses the changing nature of economic geography and how dissatisfaction with previous methods and results led to shifts in paradigm, ultimately leading to the current institutionalist approach. This thesis argues that the continuing search for the right theoretical framework in part can be explained by the failure to realise that economic geographers are dealing not only with complex systems, but with evolving systems as well. Our economic environment is an open-ended system, with constantly changing boundary conditions, and as such it should not be approached under the *ceteris paribus* assumption. The recent turn of economic geography away from a quantitative approach was, in part, an attempt to deal with this problem. However, the turn stranded the field in a descriptive approach. Descriptive approaches are not very successful in providing insight into complex systems, in that they focus on specifics of location, and fail to recognize the principle processes governing the observed dynamics.

Economics, where much of the inspiration for research in economic geography originates, faces exactly the same situation, and these issues are discussed in section 2.2. Mainstream economics is still largely the domain of general equilibrium theory. None of the often very impressive theory on the relations between prices, markets, supply and demand has anything to say about the always growing and changing economy, yet this fact seems to be ignored. Von Neumann provided an elegant theory on expanding economies (von Neumann, 1946), but expansion is limited to a multiplication of economic activity per sector by a constant factor, so although the amount of activity increases, the relations among the activities do not change, and the types of activities do not change. That is not the type of expansion this thesis addresses. Like in Schumpeter (1961, p.63) we do not consider mere growth as a process of development. Here we aim to model an ever growing set of products and technologies. The creation of new sectors, rather than the enlargement of existing sectors, is the main focus of the model here. The constant wave of new goods that flood our markets, and the disappearance of old-fashioned products that have become obsolete, is the kind of expansion we aim to understand. Schumpeter provided a description of these processes (Schumpeter, 1961). Here we go further and capture the processes in a model. This thesis argues that though this has been tried before in the field of evolutionary economics founded by Nelson and Winter, so far none of the models have been evolutionary, in the sense that none of the models are constructive, a concept borrowed from the field of artificial chemistry.

Kauffman's work on a generic biology deals with the emergence of order and structure, and in one of his books he describes a simple constructive economy (Kauffman, 1995). His Lego world idea (Kauffman, 2000), discussed in more detail later, is an appropriate scheme for a constructive system; it just needs to be modelled in an economic context. Since some of the tools developed in artificial chemistry will be useful in the context of economics as well, some of the relevant ideas are discussed. The chapter closes with an introduction to von Neumann's work on expanding economies, and follows some of the discussion his models generated.

2.1 Economic geography

Economic geography has always been searching for an appropriate paradigm. Over time, a wide variety of approaches have been explored, and until very recently, all of these approaches have been closed. In other words, the system under study is assumed to be an isolated system of which everything there is to be known is known, there is no interaction with the outside, and subsequently, one aims at giving the final destiny of the system under study. The final destiny is either a description of how we arrived at the current situation, or a stable state that will be reached when everything is in equilibrium. This section consists of two parts; the first part discusses the characteristics of economic geography that made the approaches closed, followed by a subsection that discusses a partly successful attempt to get away from this limitation.

2.1.1 The closed approach

Economic geography first emerged as commercial geography, which discussed the relations between products and location: where did products originate, and where were they going? When, in the nineteen fifties and sixties, the descriptive approach was rejected in favour of a search for a general theory, earlier German work, such as that of von Thunen, Christaller and Lösch, was introduced into the anglo-saxon research community, and became the foundation of an expanding body of location theory. This new approach was based primarily on the minimum energy, equilibrium paradigm borrowed from physics via economics, and these theories aimed to demonstrate the stable state of the system.

In the nineteen seventies, economic geographers began to realize that these models were unsuccessful in explaining spatial economic behaviour in often rapidly changing environments. Thus, when this formal theoretical approach did not satisfy, geographers changed their focus back to localities, and moved away from generalized neoclassical economics (Boschma and Frenken, 2006). This shift, around 1980, has been called the cultural or the institutional turn. Several streams of economic geography sprang from this turn, such as the Marxist approach, the structuration approach, and behavioural geography, and these can be put under the umbrella of institutionalism. According to Boschma and Frenken:

> In its most stringent form, institutional approaches argue that differences in economic behaviour are primarily related to differences in institutions. Institutional differences can be present among firms (in term of organizational routines and business cultures) and among territories (in terms of legal frameworks, informational rules, policies, values and norms). Comparative analysis between these units with different institutions can then be related to differences in economic outcomes, such as profit, growth, income distribution and conflicts. (Boschma and Frenken, 2006, p. 276).

As far as understanding evolving systems is concerned, which is the ultimate aim of this thesis, this was certainly a step in the right direction, since the institutionalist approach focused on differences in local specifics to explain differences in global developments. This is already a much more open theory, much less restricted by the constraints that come with analytical research methods.

Apparently the move away from spatial neoclassical economics created a vacuum, and in the nineteen nineties, economists moved into this void with the new economic geography (Krugman, 1995; Fujita *et al.*, 1999). Though the work by Krugman and his followers does not rely on analytical techniques to come to a result, but instead uses numerical examples through simulations, and thus appears to be dynamic, it does so only in order to determine the stable end result. It thus merely reintroduced the consummatory neoclassical equilibrium approach into the geographic domain to explain trade, specialization and agglomeration.

Though this approach adds to the existing body of neoclassic economic geography, it ignores the fact that such an approach had been tried and declared unsuccessful to explain many phenomena. By again "using formal models assuming utility maximization, representative agents, and homogeneous space and using equilibrium analysis to come to theoretical conclusions or predictions" (Boschma and Frenken, 2006, p. 277), it goes straight back to the economic geography from before the cultural turn. In addition, a very similar approach exists that predates Krugman by almost twenty years (Allen and Sanglier, 1979; White, 1977, 1978, 1990). Needless to say, such a move by economists into the domain of economic geography has not been well received, and there is little cooperation between the two subfields. As Boschma and Frenken write, "on the contrary, the debates have been fierce and with little progress" (Boschma and Frenken, 2006).

Boschma and Frenken write that the institutionalist approach and the new economic geography differ in two ways: firstly, formal modelling is absolutely not done in the former, while it is the basis of the latter, and secondly, bounded rationality and contextuality are the essence of the institutionalist approach, while the opposite is assumed by Krugman.

Boschma and Frenken detect a third, alternative, approach making its way in economic geography, and place it between the other two. This third approach is evolutionary economic geography.

2.1.2 Evolutionary economic geography

Evolutionary economic geography is spatially applied evolutionary economics, which is based on Nelson and Winter's work from the late nineteen seventies and early nineteen eighties (Dawid, 2006; Foster and Hölzl, 2004), and more distantly on Veblen, who can be considered the first institutionalist (Hodgson, 1993). Although Schumpeter's work is often mentioned as the great example of evolutionary economics (Foster and Hölzl, 2004; Andersen, 1994), Hodgson explains that this is unjust and misleading: evolutionary economics finds much of its inspiration in the work of other economists such as Veblen and Marshall and in biology, while Schumpeter himself categorically refused to use a biological metaphor (Hodgson, 1993, 1995). Regardless, Schumpeter's description of development through qualitative changes in a static system has inspired the field of evolutionary economics, where his name is found everywhere. Evolutionary economic geography is still a small field (Dawid, 2006). It focuses on dynamics by means of Lamarckian adaptation, and aims at understanding spatially changing technology. Furthermore, it does not assume the homo economicus, the representative agent; on the contrary, it assumes variety among the agents.

Boschma and Frenken argue that evolutionary economic geography can be positioned between institutional economic geography and new economic geography. It shares with the neoclassical approach the use of formal modelling, and with the institutionalists it shares the contextuality. However, it differs from both these approaches in that evolutionary economic geography is dynamic. It is not heading toward an equilibrium, and it does not explain the differences in current organisation based on the difference in present institutions. Instead it explains by studying the interaction of present routines.

Werker and Athreye (2004) also mentions the difference between the new economic geography rooted in equilibrium theory and the more process based evolutionary approaches. What is more important, however, is their discussion of the effect of knowledge and its spillovers. They mention that:

> Changes in the endowment of infrastructure and production factors might lead to changes in regional supply and demand, which in turn affect the industry structure of a region. As the regional industry structure is the outcome as well as the driving force of regional supply and demand, it is often central to the analysis (Werker and Athreye, 2004, p.513).

However, the examples of such analysis are then limited to the interaction between industry structure and local knowledge spillovers, and knowledge accumulation. Results have been sparse, and to a certain extent, contradictory (Werker and Athreye, 2004).

This thesis will show that supply and demand, i.e. the system of input and output, and thus the flow of commodities required for existing and innovative technology, together provide an easier and more interesting link between innovation and industry structure. We agree here with Metcalfe that innovation itself is unpredictable in detail: " Innovation is about surprise and surprises are not predictable" (Metcalfe, 1998, p.86). However, nothing can come of nothing. Innovations are combinations and new applications of existing concepts and commodities. By creating a constructive artificial economy, in which existing products and technologies are combined to create new products and technologies, the effect of innovation on regional supply and demand is made formally explicit and can be studied.

In order to generate such a constructive artificial economy two models need to be integrated - a model for innovation and a model of the market, which serves as a selection mechanism. Markets are difficult to model. Many attempts following the neoclassical paradigm exist, but these lack locational and individual specifics. Here an agent based model of production and trade has been developed to deal with locational and individual differences, where production and trade are regulated by a price mechanism.

In the book *Production of commodities by means of commodities* (Sraffa, 1963), Sraffa explains how it is possible to replace the neoclassical construct of utility, demand, and supply for determining price (the marginal method) by a much simpler method for determining prices in the steady state. According to Sraffa, in stable situations where coefficients, for example, do not change, the focus on change that is so much part of the study of marginal cost, is unnecessarily complicated. By analysing stable flows between sectors, it is possible to derive the prices based on comparison of input costs and output value per sector. If one assumes that the system is in a steady state, and thus, that next year's production figures will be the same as this year's, then this year's output value should enable the sector to acquire next year's input. In the study of dynamic economics, additional information is required, but still much less, than what is required for the neoclassical approach (Blatt, 1983). Thus, one does not have to make neoclassical assumptions on the character of demand and utility functions: these naturally result from the agent based simulation of a local market if one follows Sraffa's reasoning on prices. The model in this thesis does precisely that.

There is empirical support for such an approach. Essletzbichler and Rigby show that there is significant difference between regions in terms of applied technology, and this difference is persistent (Rigby and Essletzbichler, 1997, 2006). Theirs is a statistical analysis of Census of Manufactures data. It does not investigate the underlying causes of these differences, but the variety is certainly there, in support of the evolutionary economics approach and falsifying the generalizing assumptions of equilibrium theory.

Dosi *et al.* (1995) studies these regional industry systems, as does Boschma and Lambooy (1999). These approaches, especially the latter, seem rather vague and leave the true underlying process unexplained. Boschma and Lambooy study "the effect of new variety on the long-term evolution of the spatial system", but what exactly this new variety is, is unclear, and the disruptive nature of technological change is described in terms of "spatial leapfrogging" (Boschma and Lambooy, 1999, p.419). Chance is a major factor in setting up new technology in a region and after that, "chance events and increasing returns are important mechanisms to explain the spatial formation" (Boschma and Lambooy, 1999, p.424). This thesis argues that simulations are needed to test such statements.

Dosi *et al.* seem to be aware of the need to model such systems. They develop a model to simulate innovation under different technological and market selection regimes, and explain several stylized facts of industrial systems. It is important that a model of innovation include a market mechanism in order to justify entry and exit of goods and firms, and the model by Dosi *et al.* "tries to specify an explicit market dynamics" (Dosi *et al.*, 1995, p. 418) . The inclusion of a realistic market mechanism is an element of paramount importance in capturing the essence of evolving economies. The market mechanism proposed by Dosi *et al.* consists of a probabilistic rule that determines market share and death based on an artificial measure of competitiveness. The resulting model is a dynamic model that deals with innovation and selection through a market mechanism; this is done in a very simplistic way. Innovation consists of a random increase in a measure of the competitiveness of a firm, c_i. In addition, as is the case with all followers of Nelson and Winter, nothing new is added to the system, and "the representation of the process of technological change leaves a large black box between the inflowing funds and the resulting productivity increase" (Dawid, 2006, p. 1248). Since competitiveness is an artificial construct measured by a number c_i, the market and market share in the model are equally artificial.

Though such work makes a start toward investigating the consequences of a changing technology expressed by a changing competitiveness, it does not allow the study of the resulting changes in the industrial system as proposed here. The interaction between the industry structure and innovation, mediated by supply and demand, such as was argued for in Werker and Athreye (2004) and addressed in this thesis, is not studied in the neoclassic or institutional approach. Nor is it studied in evolutionary economics. Despite the focus on adaptation and a dynamic perspective, and despite the word evolutionary in the subfield's name, evolutionary economics really only deals with dynamics. The source of this shortcoming is that creativity or true innovation is lacking. As Dosi *et al.* discuss Jovanovic's work (Jovanovic, 1982), they state that "the universe of available techniques is given from the start: hence 'learning', in the sense of discovering something genuinely new, is ruled out" (Dosi *et al.*, 1997, p.16). Likewise, Nelson and Winter write:

> Use of the term "search" to denote a firm's activities aimed at improving on its current technology invokes the idea of a preexisting set of technological possibilities, with the firm engaged in exploring this set. This connotation ... seems less natural when one is considering R&D aimed to develop a new aircraft, or, more generally, R&D activities where the terms "invention" or "design" seems appropriate. Instead of exploring a set of preexisting possibilities, R&D is more naturally viewed in these contexts as creating something new that did not exist before. (Nelson and Winter, 1982, p.210)

Up until this point, the argument in this thesis is fully compatible with the stance taken by Nelson and Winter. However, they then continue the above statement with the following remark:

> But for the purpose of our evolutionary modeling, the distinction here is one of semantics not substance. The R&D activities of our firms will be modeled in terms of a probability distribution for coming up with different new techniques. We will discuss this in terms of sampling from a distribution of existing techniques. (Nelson and Winter, 1982, p.211)

Though Nelson and Winter are aware of the problem, the lack of novelty is swept under the carpet, and "something genuinely new" is ruled out. Evolutionary economics has mainly followed Nelson and Winter in this approach (Dawid, 2006). The present thesis addresses this problem in that it shows that modelling technological evolution does *not* invoke the idea of a preexisting set of technological possibilities. As a matter of fact, it is simply impossible to pre-state the technological possibilities at any given time (Kauffman, 2000). Doing so leads merely to optimizing the functioning of the institution in a given environment, which includes the so called innovations right from the start. In this case, variety is "generated" by selecting new technology from a list. However, evolution requires three ingredients (Rigby and Essletzbichler, 1997), of which one is a persistent source of variety. Furthermore, it requires some mechanism that allows the inheritance of qualities to preserve the system, and some mechanism which performs selection, be it better reproduction or better survival. True, the latter two processes are both present in evolutionary economics. The heredity comes from the copying of behavioural routines. Combined with a selection mechanism, this results in a model of Lamarckian adaptation (Rigby and Essletzbichler, 1997). Lamarck offers a theory for the inheritance of characteristics acquired during the lifetime, as opposed to Darwinian inheritable traits, which are left unaffected by life experiences. However, this thesis argues that a persistent source of novelty is missing in models based on Nelson and Winter, and thus in evolutionary economics. Other work related to technological evolution, such as work by Frenken and Nuvolari does allow novelty (Frenken and Nuvolari, 2004). However, such work depends on randomly defined fitness, and thus features a highly artificial selection mechanism. Until now, there does not seem to be an appropriate economic model that combines novelty, heredity and selection. The biggest challenge is to model technological novelty. To avoid having to specify preexisting sets of qualities of some sort, we will use constructive systems, a concept borrowed from artificial chemistry. This approach will be discussed in Section 2.4 of this chapter.

Most of the papers discussed in this section are from the last two decades. This suggests that evolutionary economics is new. This is not the case however. Long term economic development and change have been considered important since the early days of modern economics. To what extent it has actually been studied is explored in the next section.

2.2 Long term behaviour, dynamics and evolution in economics

2.2.1 Mechanical economics

In this section we explore what economics has to say about whole economic systems. Most of the topics discussed in this section will be used in some form later in the thesis. Much of the discussion here is based on Canterbery (2001), who provides a chronological treatment of the history of the "dismal science", economics.

In the *Tableau Economique* of 1759, Quesnay and the physiocrats argued that the economy functioned like the circular flow of blood through a body. Agricultural production led to a surplus, the output being higher than the production costs due to the added value of the use of land, which was a gift of nature. Only agriculture could generate a surplus due to this added value of the land. The surplus of goods from the land flowed to other sectors. These sectors merely transformed the goods: there was no added value, because manufacturing output equaled input. Labour was paid for by wages, wages were used to purchase goods, which enabled the labour force to work. This created a circular flow of goods, and an opposite flow of money in the form of purchases and wages. However, a large part of the money flow was diverted to land holders who received money, without producing anything, in the form of rent. Even more surprisingly, taxation was based on production and not on possession of land. According to Quesnay, the wrong sector was taxed, and restricting the circulation by taxation or hampering the flow by tariffs would slow down the long term development of the system. Therefore, Quesnay argued for free trade instead of mercantilistic tariffs. This metaphorical use of the cardiovascular system to model the flows through the economy is a very interesting idea, and it is one that was too easily forgotten. Quesnay and his use of the circular flow were later revived by Schumpeter and Sraffa (Sraffa, 1963; Blatt, 1983).

Adam Smith also argued for unrestricted economic activity and saw a much bigger role for manufacturing surplus in the economic system. In the post Newton era, he argued that once the economy was set in motion by the hand of God, there was no need to interfere. Self interest drives the system, but self interest also is held in check by the self interest of others. "An individual is led by an invisible hand to promote an end which was no part of his intention" (Canterbery, 2001, p.54). Through prices, the laws of the market determine the quantities of goods produced, as well as the income of workers and manufacturers, and there is no need to interfere in order to achieve the optimal state. Again, the economy was treated as a whole, where local micro-scale decisions generate a well functioning macro-economy. Hodgson (1993) speculates on Adam Smith's influence on Darwin, particularly the idea that the interaction of individuals leads to a larger, more complex structure that comes with a natural organisation, regardless of the fact that none of the individuals is concerned with anything other than his own situation. This might have inspired Darwin to realise that the sum of all the parts can result in more than a mere collection of the parts.

During the industrial revolution in England, with a population explosion and rapid urbanization underway, Malthus had ideas similar to those of Smith: do not interfere with the system (i.e. no relief for the poor), because it will only make the problem of paupers bigger and therefore impede the development of the system. Furthermore, Malthus' description of the struggle to survive was picked up by Darwin, who wrote about it in his notebooks (Hodgson, 1993). Apparently, in the nineteenth century there was much interaction between biology and economics. Toward the end of the nineteenth century, Veblen applied elements of Darwinian evolution to economics, beginning a new strand of economics, the work of the institutionalists. This has never been mainstream economics (Boschma and Frenken, 2006) but it did go through a revival when Nelson and Winter picked it up in the nineteen seventies and eighties. However, mainstream economics followed not biology, but physics (as did economic geography).

Ricardo, a contemporary of Malthus, argued once more for putting an end to tariffs, which were lobbied for by a still powerful class of landowners. For the economic system to perform well, the organisation of the system should be left to the free market. In the welfare economics of Smith, Malthus and Ricardo, liberalism plays a major role. Since manufacturing was the source of development, nothing should be done to impede manufacturing, in order to generate as much activity as possible. Landholders should be taxed instead of being protected by tariffs. Ricardo developed a model of relative prices to illustrate how the artificially high price of corn, and thus labour, would hamper industrial development and favour landed aristocracy. Later, Sraffa expanded Ricardo's system of relative prices. Sraffa's theory of value will be used in the model developed in this thesis. The value of commodities, in Ricardo's system, depends on the costs of the products and labour involved in producing the good. This is key in the classic approach.

Marx wrote that classic liberalism would not save the system. Capitalism and the free market depend on healthy competition to function well, but the same competition drives competitors out of the market until only monopolies remain, and thus, capitalism is destined to collapse. His model of the economic system showed an unavoidable revolution of the economic system. In many ways, this resembles the description of an evolving system, but Hodgson argues that the developments in Marx's theory are of a different kind than those in Darwinian evolution. Hodgson writes that the class struggle in which individuals are "collectively engaged" is very different from the race between a wide variety of individuals in Darwin (Hodgson, 1993). Furthermore, Marx saw the socioeconomic system evolve to an equilibrium state, i.e. the classless society. One could say that Marx's theory was much more mechanical than evolutionary.

The neoclassical school of thought differs from the classical through the development of an alternative theory of value. Whereas the classic idea of value was based on the cost of production, the neoclassical approach based value on utility. Utility is difficult to quantify, but Marshall provided a solution to this problem by linking price and utility; if one is willing to pay more, utility is higher (Canterbery, 2001, p.132). This idea was coupled with marginalism both on the demand and the supply side. Incremental cost and incremental utility are compared by the consumer and at the same time marginal cost is determined by the producer. The price then follows from a balance between marginal utility and marginal cost, and supply and demand match at that price. In addition to the treatment of land (Quesnay) and labour (Smith) as production factors, the neoclassics introduce capital as an essential ingredient.

Marshall provided a mathematical method to value commodities one at a time by comparing utility, demand and supply. If intermediate products involved in the production increase in price, production costs change and a new balance between supply and demand has to be struck, which in turn would affect other prices. Walras developed a theory of value that dealt with all commodities at the same time. He developed a system of equations to set the relations between production and prices and purported to demonstrate that there was an equilibrium for the whole economic system by showing the number of variables to be equal to the number of equations. However, this is neither a necessary nor a sufficient condition for the existence of a solution. Wald for the first time provided a set of conditions and a rigorous proof of the existence of an equilibrium solution by allowing inequalities (supply should be greater than or equal to demand). This implies that some products can be overproduced, with price zero, while others will have positive prices, and likewise, some production processes are used while others are abandoned (Morgenstern and Thompson, 1976, p.2). Hicks based the set of equilibrium prices on the behaviour of consumers and producers (Canterbery, 2001, p.137). Arrow and Debreu subsequently used set theory and the Nash equilibrium in order to prove the existence of equilibria (Canterbery, 2001, p.406). For these equilibria to exist under conditions of uncertainty they did however require perfect foresight, which means that every producer and consumer has to know all possible future price sets and production quantities. Ultimately, the theory of value and pricing has become very impressive.

but very complicated and abstract as well. Concepts such as utility and demand are difficult to capture, except in a very simplified form, and the non-economics community has always been reluctant to accept economists' assumptions on these matters.

An alternative has already been mentioned, namely the classic theory of value based on cost of production. Leontief and Sraffa expanded this theory by taking into account direct and indirect links between sectors. This approach is dealt with in the next chapter.

Canterbery mentions that Marshall's name has become synonymous with neoclassical economics (Canterbery, 2001, p.122), while Marshall's work on the evolution of economic institutions was ignored. "His followers chose to develop only Marshall's analytic footnotes" (Canterbery, 2001, p.141). Hodgson notes that, although Marshall's followers were most definitely neoclassic economists, Marshall's work contains references to so called equilibrium states being transitional (Hodgson, 1993, p.105), and not necessarily an optimal, perfect equilibrium. Furthermore, Hodgson detects aspects of Lamarckian evolution in Marshall's later work. These are exemplified in the following passage from Marshall, all apparently left unnoticed by his followers:

> The catastrophes of mechanics are caused by changes in the quantity and not in the character of the forces at work: whereas in life their character changes also. 'Progress' or 'evolution', industrial and social, is not mere increase or decrease. It is organic growth, chastened and confined and occasionally reversed by the decay of innumerable factors, each of which influences and is influenced by those around it; and every such mutual influence varies with the stages which the respective factors have already

> reached in their growth. In this vital respect all sciences of life are akin to one another, and are unlike physical sciences. And therefore in the later stages of economics, when we are approaching nearly to the conditions of life, biological analogies are to be preferred to the mechanical, other things being equal (Hodgson, 1993, p.106).

As has been mentioned before, the problem with equilibrium models is that they are not even an appropriate vehicle for studying dynamic systems, much less evolving ones. An example is our atmospheric system (Blatt, 1983). Due to the constant flow of energy into the system, the resulting state of our atmosphere is never in equilibrium. Arguably, the boundary conditions of our weather system are stable, but nevertheless, every day we experience very dynamic, and far from equilibrium weather. Our economy sees a similar flow of energy fueling the system (the same sun, or indirectly, fossil fuels, raw materials etc.). In addition, the interactions among products of the economic system keep changing and the economy grows and becomes more complex. We started with a simple economy of beads and primitive tools made out of stone, and we now trade work at a desk for iPods. The variety of products, the technological capabilities, and the many forms of work that are performed have all increased tremendously. Naturally, there is a drive for demand to meet supply: we need to balance expenditures with income. However, this needs to happen under always changing circumstances. Therefore, the *ceteribus paribus* equilibrium approach does not provide us a complete description of the economic system. We need to look further.

Alternatives to the static Walrasian system have already been developed. The first alternative was descriptive, as Schumpeter developed a theory on the role of the entrepreneur and innovation in the economic system (Morgenstern and Thompson, 1976). Schumpeter's model of economic life is based on a description of a circular flow of goods, consumables and land and labour. This is reminiscent of Quesnay's use of a the cardiovascular system to illustrate the functioning of an economy. Decisions regarding production (to produce means to combine materials and forces (Schumpeter, 1961, p.65)) and trade are made with the help of data and experiences from the past. Under constant conditions (no sudden changes in the data) and assuming that one strives for making the best combinations of available resources, gained experience leads to the development of well-tried routines and Walras' stable state.

On top of this static circular flow model Schumpeter places a model of development. For Schumpeter, development means a change in possible combinations: a new product, a new production method, opening of a new market, a new source of supply, a new organisation. However, the appearance of a new combination is only the invention. Successful introduction of the invention required two additional actions. The new combination is based on existing goods. Since all existing goods are produced according to the needs in the existing stable circular flow, and similarly finances are only sufficient to supply the existing system, credit in order to acquire the required resources has to be provided from somewhere else, and this is the task for the capitalist. Next the new technology has to be embedded in the existing system. Since economic activity is based on data and experience, and this is non-existent for the new technology, a persistent visionary, the entrepreneur, is needed who can guide action without falling back on previous experience. Here Schumpeter provides his take on the actors of the capitalist economic system: the manager dealing with every day routines in the circular flow model, the inventor-capitalist-entrepreneur team supplying the development. Most, or maybe even all of the post-schumpterian contributions in evolutionary economics have dealt with the role of the entrepreneur-capitalist combination. What is the effect of different market regimes on innovation, and can adaptive R&D strategies result in an evolving economy? The part played by the inventor, namely the invention of a new combination to create a new product or a new technology and its effect on the economic flows has been ignored. This is exactly the point made by Werker and Athreye mentioned earlier.

More or less at the same time as Schumpeter wrote his theory on economic development, von Neumann developed a quantitative model of general economic equilibrium (von Neumann, 1946), which was, in a very limited way, not static, but allowed balanced growth. It is possible to grow and to be in an equilibrium state at the same time. This approach, discussed in more detail below, perfectly models Schumpeter's circular flow description of a static economy. In addition, it turns out to be a very useful basis for modelling Schumpeterian development as an expanding evolutionary system, as this thesis will show.

According to Hodgson, Schumpeter's work is inconsistent since it is an attempt to unite the homeostatic Walrasian system with dynamics that depend on fleeting elements which need to be ruled out in Walras' work, and it does not serve as a basis for theory (Hodgson, 1993). According to Hodgson the drive for stability required to explain the existence of a stable economic system cannot be combined with the drive to disrupt this system by introducing changes to the system. This thesis disagrees with Hodgson on this point. Whatever the motives of inventors, capitalists or entrepreneurs for economic actions are (Schumpeter, 1961, p. 93), why should they be the same as those of the managers? In addition, the model developed in this thesis shows that striving for maximum personal gain given the local circumstances can lead to stable behaviour as well as innovating behaviour, and in that case managing agents and innovating agents act on the basis of similar rules.

To conclude this description of mechanical economics, twentieth century models of economics still dominate economics in the twenty-first century. These models are based on neoclassical equilibrium theory and do not have much to say about dynamic systems, much less evolving ones. At best, they deal with growing systems. These systems grow quantitatively, not qualitatively, and Marshall's remark on the importance of being able to deal with qualitative changes is more valid than ever. For example, the turnpike theories based on von Neumann's constant growth rate projections, which describe the fastest way of economic quantitative development i.e. the largest expansion factor, are the closest the neoclassical approach gets to modelling long term behaviour of the economic system. As a reaction to this, a new approach, evolutionary economics, appeared in the nineteen seventies and eighties. This approach builds on the nineteenth century institutionalist economics by Veblen and others. The following section focuses a little more on the problems addressed by this new field.

2.2.2 Evolutionary economics

Dawid (2006) distinguishes a number of important aspects of innovation that have been incorporated into evolutionary economics, and which have been ignored by traditional economics. One such aspect is the relation between research and development and knowledge accumulation and spillover. Another aspect is the prediction, by means of agents' internal models, of the outcome when introducing new technology. The performance of new technology is very difficult to predict. However, in order to assess the profitability of new technology, estimates regarding anticipated effects are very important for firms. A third aspect dealt with in evolutionary economics is the importance of the heterogeneity of adaptive strategies, i.e. a firm's innovative strategies. In addition to the inclusion of these facets of innovation, which would be hard to achieve using the analytical approach, evolutionary economics has focused on generating so called stylized facts (Dosi *et al.*, 1997). Examples of stylized facts are

- stable power law distribution in firm size
- persistence of asymmetric performances
- instability in market size and market shares
- turbulence in entry and exit of firms
- life cycles of products

While these phenomena are left unexplained by the neo-classical approach, evolutionary economics uses models to explain the stylized facts by experimenting with the conditions under which the models generate them. This can be done for each fact at a time, but it is even more important to combine all of these puzzles and find an explanation for these facts simultaneously (Dosi *et al.*, 1997). How can we obtain stable firm size distribution with a persistent variety of performances within one sector, while products come and go, firms come and go and markets come and go? Evolutionary economics is gaining momentum and adding to the understanding of economic systems. However, as argued in section 2.1.2, such models have to be made constructive in order to deal with innovation and evolution.

It is interesting to see how economics has moved away from biology. While in the nineteenth century there was a certain influence of Smith and Malthus on Darwin (Hodgson, 1993, p.70), and in return, of Lamarck, Darwin and Spencer on Veblen and Marshall (Hodgson, 1993, p.81), economics, and in particular neo-classical economics, moved further away from biology during the twentieth century. Hodgson's *Economics and Evolution: Bringing Life Back into Economics* argues for economics to find inspiration in the field of biology (Hodgson, 1993). Kauffman, originally a biologist, argues that frameworks that apply to biospheres apply similarly to econospheres (Kauffman, 1995, 2000), and also argues that "economics should shift its attention from physics to biology" (Kauffman, 2006). We can find some useful elements in Kauffman's work, and in artificial chemistry, that need to be inserted into current evolutionary economics to make it truly evolutionary.

2.3 Artificial biology

Investigations is Kauffman's latest book and it is this book that started this thesis. Kauffman describes his search for a general biology, not the specific biology here on our planet, but the general rules that govern evolution. Kauffman is mostly known for his work on evolution and in particular, NK fitness landscapes (Kauffman, 1993). That particular model has been applied to study the structure of innovation of technology independent of economics (Kauffman and Macready, 1995), and a generalisation of the NK model by Altenberg (Altenberg, 1997) has been applied to technological innovation in Frenken and Nuvolari (2004).

As is discussed in Fontana and Buss (1994) and in Kauffman (2000, p.16), Darwinian evolution, that is, the interplay of reproduction with mutation and recombination together with selection, is not sufficient to explain the emergence of complex structures. There first have to be structures that are capable of reproduction. Therefore, Darwinian evolution by itself cannot have led to the formation of the first self-reproducing structure, simply because it requires reproduction to do so. Evolution alone, therefore, is not the complete story. In addition we need emergence. But emergence of what? There is a large literature on the origin of life; see for example the work of Hazen (Hazen, 2005). In *Investigations* Kauffman marvels at the emergence of something that can act on its own behalf, a union of matter, energy and information, an autonomous agent. As part of an answer to the question of how such a thing can emerge, he proposes:

> in an auto-catalytic set, all the molecules whose formation must be catalyzed find molecular species within the set that catalyzes the reactions forming each of those molecules (Kauffman, 2000, p.32).

The auto-catalytic system is Kauffman's solution to the emergence of reproducing systems. It is possible that the first self-reproducing systems were based on RNA or DNA structures, but this is not very likely, since the building blocks and catalysts required in this process are produced by executing genetic instructions. Kauffman argues that it is much more likely that auto-catalytic sets were the first self-reproducers; he shows that the emergence of auto-catalytic sets is very likely. In addition, a metabolism has to emerge to solve the thermodynamic problem (Kauffman, 2000, p.47). The linking of building blocks requires energy, which needs to be generated. If the generation of energy is cyclical, this process can be repeated. For

Kauffman, the two components, reproduction and metabolism, are the essence of life and autonomy:

> An autonomous agent must be an auto-catalytic system able to reproduce and able to perform one or more thermodynamic work cycles (Kauffman, 2000, p.49).

Matter is available in many parts of the universe, as is energy, since the universe has been out of equilibrium since the big bang. These deviations from equilibrium can be exploited to extract work, and this work will lead to new, different displacements from equilibrium, that can be detected again, and utilized in some other, slightly more complex, way. Kauffman believes that auto-catalytic systems are capable of doing this. In short, Kauffman proposes a theory for the persistent emergence of new species in an ecosystem and the extinction of old species.

> But the econosphere has similar extinction and speciation events. Consider my favorite example: The introduction of the automobile drove the horse, as a mode of transport, extinct. With the horse went the barn, the buggy, the stable, the smithy, the saddlery, the Pony express. With the car came paved roads, an oil and gas industry, motels, fast food restaurants, and suburbia. The Austrian economist, Joseph Schumpeter, called these gales of creative destruction, where old goods die and new ones are born (Kauffman, 2000, p.216).

Such a system, in combination with Darwinian evolution, could very well be a good model for an evolving economy.

2.3.1 Lego world

In Kauffman (2000), but also in Kauffman (1995), Kauffman describes an idea for a model of an evolving economy. As in molecular biology, different building blocks are combined into new things, and sometimes these turn out to be very useful. An analogy that Kauffman uses is the Lego world. Smaller Lego blocks are combined into new building blocks, and as a result, the Lego world moves into the adjacent possible, i.e. the collective of concepts candidate for incorporation into the next level of a system. Kauffman shows this applies to the biosphere, and argues it applies to the econosphere as well. This thesis shows that the same ideas can be applied to the econosphere, and through simulation it is possible to learn about systems like these. The proposed model is based on Kauffman's description of the econosphere moving into the adjacent possible, combined with a market mechanism to make decisions about extinction, survival and success.

In real life, product innovations are new consumption goods, new intermediates, or new production techniques. These are merely combinations of already existing products, or new ways of combining existing products, an example of which is shown in Figure 2.1. This figure shows that a mobile phone, an iPod, a portable hard drive and a camera are easily combined to create a new product, called an iPhone, and consumers are lining up to obtain one. Such new combinations are not only a recent phenomenon. A water pump driven by steam power and the rotary motion of a waterwheel were combined to create the first steam engine. This round-about way of generating rotary motion by means of a water pump driving a waterwheel was applied first, and even preferred over a steam engine directly driving a shaft, because of the steady regular power provided by the flow of water and the buffer capacity of a small reservoir, which could keep the mill going when the pump temporarily stopped (Hills, 1970, p.135). The successors of this steam engine, engines driving a flywheel through the use of a crank, allowed a major reorganisation of the economic system. Many more examples, such as the kinetoscope, composed of a light bulb, celluloid (derived from the production of billiard balls), and the zoopraxiscope (which provided the principle of generating the illusion of movement based on static images), can be found in Burke (1978). Burke is currently preparing a website (Burke, 2008), which allows the user to explore the technology connection graph.

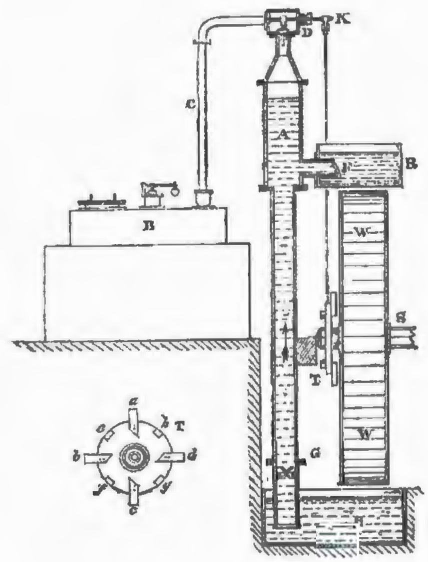


Figure 2.1: Innovation now and in the past. The left figure (Apple, 2007) shows an iPhone, which combines the functionality of a phone, an iPod, a portable hard drive, a photo camera and a touch pad screen. These elements combined create a hot new consumable. The right figure (Farey, 1971, p.123) shows the design of an early steam engine. It combines a Savery steam pump with a waterwheel to generate rotary motion. Later the waterwheel was replaced by a rod and crank construction which had long been known from, amongst other things, spinning wheels.

In economics, as in biology, auto-catalytic cycles play an important role. The high demand for coal due to the shortage of wood in sixteenth century England led to larger mining operations, that operated at greater depths, requiring the use of much metal machinery and tools. The increased need for iron to produce these tools led to a higher demand for metal parts in the smelting operation, as well as a greater need for coal to fuel the blast furnaces. Transportation involved in supplying both metal and coal required, even in the age of wooden ships, many metal parts, as well as cannons and sheathed hulls for protection (Nef, 1936). Just as Eigen (1992) argues that auto-catalytic sets will outperform non auto-catalytic sets, so it can be argued that England outperformed France, and experienced a much faster growth due to the effects of positive feedback in the cluster of metallurgical and mining industries. (See also Padgett *et al.* (2003) for a model that simulates the appearance of auto-catalytic sets in a production system.)

This thesis does not attempt to add anything to the study of the evolution of life. However, the idea is that theories on the evolution of life and the emergence of structure, developed in biology, particularly molecular biology, artificial life, and artificial chemistry, can be applied to economics as well. This is not a new idea as Padgett *et al.* (2003) shows, but the idea has never been fully developed and implemented. Kauffman talks about the econosphere as a system similar to the biosphere when regarded as a grammar or a rewriting system. These grammar systems, which give rules for performing replacements, are studied in artificial chemistry.

2.4 Artificial chemistry

As mentioned above, Darwinian evolution does model an adaptive process, but it does not explain the emergence of life. The appearance of the reproducing entities that are required for Darwinian evolution itself cannot have happened through the Darwinian process. Because of this, much research in artificial life and artificial chemistry aims at explaining the emergence of organisation in general, and in particular, the emergence of systems capable of self-maintenance and self-construction. Artificial chemistry investigates such systems by modelling the interaction of artificial chemicals and studying under which conditions certain structures emerge.

Dittrich *et al.* (2001) provides an overview of the literature on artificial chemistry and they classify the different models used in the field. Their description of constructive artificial chemistries fits perfectly what this thesis purports to demonstrate for evolving economies. The model in the next chapters will be an example of an explicit constructive dynamic system. This section will explain what is meant by that, and it will briefly discuss some models that resemble what is done in this thesis.

2.4.1 Molecules, reactions and constructive systems

An artificial chemistry can be defined as a triple (S, R, A), where S is the set of all possible molecules, R is a set of collision rules, and A is an algorithm describing the domain and how the rules are applied to the molecules.

The following example from Dittrich *et al.* (2001) illustrates this formal definition of an artificial chemistry. Let $S = \{2, 3, 4, \ldots\}$ be the set of all natural numbers greater than 1. The set of collision rules is given by $R = \{s_1 + s_2 \rightarrow s_1 + s_3 : s_1, s_2, s_3 \in S \wedge s_3 = s_2/s_1\}$. This rule set implies that when two molecules s_1 and s_2 collide (the + is not the arithmetic addition, but is instead used to represent collision), the result of this interaction is s_1 and s_3 in case s_1 divides s_2. s_1 can be regarded as a catalyst. When s_1 does not divide s_2, the collision is elastic and no reaction takes place. Finally, the reaction algorithm A is given by a random draw of two molecules from the set S, and replaces one of them if it can be divided by the other (Dittrich *et al.*, 2001, p. 233). At the beginning of a simulation, S is a finite set of randomly drawn natural numbers of size larger than one, and during the run of the model new molecules are introduced into the model. For example, if $S = \{5, 12, 20, 28, 32, 44\}$, and 5 and 20 collide, then a new element, 4, replaces 20 and $S = \{4, 5, 12, 28, 32, 44\}$. Before this reaction took place, all collisions with elements of $\{12, 28, 32, 44\}$ were elastic. With the creation of a new catalyst 4, new functionality is introduced into the system, and many more molecules can be decomposed.

In this model, the molecules are defined implicitly: not all molecules are specifically mentioned, and all that is known is that they belong to the set of natural numbers. It is even possible that new molecules appear. However, when defining an artificial chemistry, it is also possible to explicitly define the molecule set, such as in $S = \{timber, brick, iron\ ore, wool, wheat, gold, road, village, town\}$. These "molecules" are copied from a board game *The Settlers of Catan* (Teuber, 1995), and the rules of the game state that two pieces of timber and a brick can be exchanged for a road. The collision rules for this game represent the technology present in the game, and describe the possible transformations of input into output. Another rule states that one village, three units of iron ore and two units of wheat can be exchanged for a town. The board game is a clear example of a system with not only explicitly defined molecules, but also explicit collision rules. The example borrowed from Dittrich *et al.* (2001) uses an implicit definition of the rules. The rule makes use of the structure of the natural number set, and one rule defines all reactions for all molecules in S.

These two examples of artificial chemistries illustrate three interesting characteristics. First, artificial chemistries can have various degrees of abstractness. In general, if molecules in S resemble real chemicals, the model is called analogous, otherwise it is called abstract. The number based chemistry of the first example is a typical example of an abstract artificial chemistry. In the second example, though the board game model molecules are not mirroring chemicals, the model is quite "realistic" and models economic interaction as opposed to chemical interaction. This suggests that artificial chemistry provides appropriate frameworks to model economics, and this has also been suggested in Padgett *et al.* (2003).

Secondly, the reaction algorithm can harbour a spatial topology. In the case of the replacement of a village by a town, the rule specifies that a particular village on the board can grow into a town only at that location. Therefore, the reaction algorithm specifies that when the appropriate resources are present in one location the particles can react according to the rules.

The natural number chemistry has an important third characteristic: during a simulation, it is capable of generating new elements which are added to the set S. This brings us to the following definition:

> In a *constructive dynamical system* new components can appear, which may change the dynamics of the system. This is different from a conventional dynamical system where all components and interactions are given at the outset of the process (Dittrich *et al.*, 2001, p. 235).

It is this qualification in particular that is interesting in the context of this work, since this thesis models a diversifying spatial economy, with a growing number of products and technologies. The two examples above suggest that artificial chemistries can be constructed such that the reactor algorithm is spatial, the economy can be regarded as an artificial chemistry, and most importantly, the system can be constructive in order to accommodate new products and new technology. Instead of the division of numbers by numbers in order to generate new products, new products will be generated by a mechanism similar to Kauffman's Lego-world model. New products are composed of two or more existing products. New technology will provide reaction rules of how to use the new products.

In Dittrich *et al.* (2001) a further specification is made regarding the construction of new components. A constructive system is either weakly constructive or strongly constructive. Weakly implies that new components are generated randomly, while strongly implies that new components are generated through the action of other components. Since in the model developed in this thesis, the new products are the result of combining pre-existing products, and since their introduction is dependent on the functioning of the market, we are dealing here with a strongly constructive system. However, there is also a certain level of randomness involved, as we cannot pre-state all the traits of an object, even when we know what the object is (Kauffman, 2000), and this becomes even more impossible when we deal with artificial objects. Kauffman mentions Darwinian pre-adaptations, or, in Stephen J. Gould's terms, "exaptations". Others have called it evolution by side-effect. The flying squirrel had ugly skin flaps that had no function until the squirrel flew away to escape a predator. The tractor engine was so heavy that it crushed every chassis it was mounted on, until it was discovered that the engine was actually strong enough to serve as part of the chassis itself. Such traits of products are only functional in specific circumstances, and Kauffman believes that it is impossible to pre-state all traits for all circumstances. This is taken as fact and left for what it is in this thesis, and new functions of products, both existing and new, appear more or less randomly.

Very interesting in the context of this thesis is work done by Jain and Krishna. The remainder of this section on artificial chemistry will briefly discuss their models and results, as well as ways in which their approach differs from what is proposed here.

2.4.2 The evolution of autocatalytic networks

Jain and Krishna (2003) poses two questions concerning the origin of life. First, how did the network and its processes and spatial structures come into existence when none was present? Secondly, the set of molecules involved in these structures is a very special, small subset of a very large set of possible molecules, and equally specialized is the set of their interactions. What are the mechanisms that can create such highly structured or ordered organizations? Jain and Krishna remark that similar questions are relevant for economics and social networks.

The model with which they propose to study these questions is a dynamical system in which the graph describing the network is also a dynamical variable. However, they do not treat the complete reaction graph of input and output of reactions, but merely deal with the catalysts required for the reactions. In doing this, they follow, among others, Eigen and Kauffman who both consider auto-catalytic sets as an essential part of the appearance of self-replicating systems and the origin of life. Suppose C is a directed boolean graph that has $c_{ij} = 1$ if node j is a catalyst for node i and $c_{ij} = 0$ if not.

By means of Perron-Frobenius theory on eigenvalues and eigenvectors, it can be determined if the network contains auto-catalytic sets (ACS) and, more specifically, which one is the dominant ACS. This concept of dominance goes hand in hand with the definition of the population dynamics

$$\dot{x}_i = \sum_{i=1}^{s} c_{ij}x_j - x_i \sum_{j,k=1}^{s} c_{kj}x_j$$

where x_i is the proportion of the population in state i and s is the number of nodes. Basically these dynamics imply that the change in the concentration i depends on the flux from states j to i and back. However, it is exactly this definition of population dynamics that makes the ACS determined by Perron-Frobenius the dominant one, and because of this the others can be ignored. This is very convenient, because in general, it is not easy to detect auto-catalytic sets in networks, (see section 6.3). The fact that some, or even most, ACS are not detected is no problem here, since the dominant ACS will attract the whole population. With market determined dynamics, as we will see in the next chapter, the theory in this paper loses some of its power, but still it is useful, since at least some ACS can be found. Furthermore, in economic systems, it is interesting to detect closed auto-catalytic sets, i.e. clusters of technology that are relatively self sufficient. Comparable to encapsulating clusters of molecules and reactions into compartments (Eigen, 1992), such sets could serve, for example, as plans for companies, and this would allow modelling the emergence of higher level economic organizations.

The implementation of the mechanism that makes a system constructive is not unlike the dynamics proposed in this thesis' economic model. Jain and Krishna randomly remove one of the nodes, (which means they remove column i and row i when node i is taken out of the system) and replace both with a random boolean vector of length s. According to the classification of Dittrich *et al.* (2001), Jain and Krishna's model is thus a weakly constructive implicit dynamical system.

With their model, Jain and Krishna have derived a number of interesting results on population dynamics (Jain and Krishna, 1998, 2002), and also on the entry and exit behaviour of species, which will be studied here as well, but in an economic context.

2.5 Expanding von Neumann technology

Von Neumann is in economics mostly known for his game theory (von Neumann and Morgenstern, 1944). Another important contribution to economics was proving the existence of a general economic equilibrium (von Neumann, 1946). Earlier proofs of existence consisted of Walras' counting of equations and variables. Since there are possibly more production techniques than products, "counting of equations is of no avail", writes von Neumann. Like Walras, Pareto and Wald before him, von Neumann modelled a whole economy. His predecessors aimed at proving the existence of a stable state, which would explain the existence of a clearing market and stable prices and a corresponding production system. Marx, and later Schumpeter, added an evolutionary element to the study of economics, but these additions were mainly in a verbal form and were not quantified. Von Neumann finally modelled an economy capable of economic growth, and proved the existence of an equilibrium using Brouwer's fixed point theorem. The economy can expand at a uniform rate, "without change of structure". The next chapter will explain von Neumann's model in more detail, but in this section we will discuss the role von Neumann's model has played in economics. Therefore, the model is briefly introduced here.

The economy consists of a set of n products, and a set of m technologies. The technologies describe how products are manufactured. Products are produced out of other goods, the technologies consist of input vectors and output vectors of length n, and the elements of these vectors specify how many units of each of the goods in the economy are required as input or are expected as output. For example, in an economy with $n = 4$ products (a, b, c, and d) a technology to create product d out of two units of product a, one of product b, and one of product c, required as capital, is represented in the model by input vector $\{2, 1, 1, 0\}$ and output vector $\{0, 0, 1, 1\}$. Furthermore, a constant return to scale is assumed: to generate two units of d, twice the input vector is required. Note that product c is both input and output of this technology. This is one way to represent capital in such a model. Furthermore, it also shows that joint production is possible: one technology can have multiple products as output. All m input vectors collected as columns determine the input matrix i, and the m output vectors give the output matrix o.

Let $z = \{z_1, z_2, \ldots z_m\} \geq 0$ be the annual activity levels for the m different technologies. The right multiplication of input matrix i with activity vector z gives the total input $i \cdot z$ required for the annual production. Similarly, the total generated output is equal to $o \cdot z$.

Similarly, let $p = \{p_1, p_2, \ldots, p_n\} \geq 0$ be the price vector for the n different products. The left multiplication of input matrix i with price vector p gives the input cost $p \cdot i$ for each of the technologies. The output costs are given by $p \cdot o$.

As mentioned above, von Neumann assumed a uniform expansion rate α. This means that with current annual activity levels z, next year's activity is αz. In order to maintain a sustainable economy, the annual production has to be greater than or equal to next year's input required for activity αz

$$o.z \geq \alpha i.z$$

In addition, prices have to exist such that the value of output is larger than the cost of input plus interest. When the interest rate is r and $\beta = 1 + r$

$$p.o \geq \beta p.i$$

Thus, the model consists of input and output matrices i and o and numerical unknowns z, p, α and β, with relations. Von Neumann's publication "A Model of General Economic Equilibrium" (von Neumann, 1946) gives the conditions under which a solution exists.

Von Neumann's model provides a very elegant representation of the production and pricing system. Not all of the required assumptions are very realistic though, and immediately after the article was published in English, (first publication was in German (von Neumann, 1937)) it received comments on how unrealistic it was that all production techniques require all of the products (Champernowne, 1946). Kemeny *et al.* developed an alternative set of conditions that solved this issue (Kemeny *et al.*, 1956). They replaced von Neumann's assumption by the following three assumptions:

1. Every production technique requires at least one input
2. Every product is produced by at least one production technique
3. At least one product has a price larger than zero

It is still required that products are involved in the production of other products, but not necessarily directly.

In addition to the proof of existence of an activity vector, price vector and growth and return rate, other work focused on the calculation of the unknowns. Hamburger *et al.* developed an algorithm to determine the minimum and maximum of the expansion rate α (Hamburger *et al.*, 1967). Gale showed that in order to determine a solution no more than n processes are required for a solution when n is the number of products and the number of processes m is larger than n (Gale, 1956). Weil turned this into "An Algorithm for the von Neumann Economy" to determine all unknowns z, p, α and β (Weil Jr., 1964). This involved selecting n out of m processes in order to change the problem into an eigenvalue problem. When m and n get larger, this becomes rather cumbersome, and so Thompson developed an alternative (Thompson, 1974). His algorithm has been implemented for the analysis of the artificially generated economies of this thesis in Section 6.3.

The solution to the above von Neumann technology model with maximum growth rate α and corresponding activity levels z and price vector p is often called the turnpike, the fast lane for growth since α has been maximized. The turnpike theory explains that optimal growth can be achieved by getting onto the turnpike, meaning that if activity levels do not correspond to z they have to be adjusted, even if that means temporarily lower production numbers (Dorfman *et al.*, 1958).

Regardless of the success of balanced growth theories, equilibrium models are not really appropriate for studying essentially dynamic systems, or worse, evolving systems. Note that when one speaks of expanding economies here, it implies economies with an expansion factor $\alpha \geq 1$, not an economy with innovation and diversification. There is however another way. It has been suggested by several, but has not been implemented so far. In 1956 Kemeny *et al.* wrote:

> It is interesting to note that if new processes are added to the economy, which can be done by adding new rows to A and B and hence to M_α, the decomposition as described above may change. ... Analogously, if a new good is added to the economy, which can be done by adding new columns to A and B and hence to M_α, then again the decomposition may change. ... Other more complicated changes can also occur. (Kemeny *et al.*, 1956, p.128)

Then, twenty years later, Morgenstern and Thompson published *Mathematical theory of Expanding and Contracting Economies*, and Chapter 8 discusses the possibility of enlarging small rural economies by adding rows and columns to construct an urban economy. This was done not so much for the illustration of innovation but to compare an economy with its ancestor long gone. However, section 8.3 is devoted to technological change and looks at the effects of changing coefficients on the expansion rate. The whole section consists of less than two pages, and finishes with:

> The above discussion of technological innovation in expanding economy models is only a sketch and far from complete. We hope to take up this point again at some later occasion (Morgenstern and Thompson, 1976, p.138).

It is remarkable that an idea with so much potential has never been developed. In 1983 Blatt writes:

> What about technological progress? This can be included by assuming that the list of activities $m = 1, 2, \ldots, M$ is not final, but new activities

> may be invented and hence become available for use, as time goes on. This makes the total number of processes a function of time: $M = M(t)$. There is no need to remove obsolete processes from the list, since such processes may be run at zero activity level. Although this way of handling technological progress exists in principle, we are not aware of any actual theoretical work making use of this idea.... The inclusion of technological progress appears to us to be a highly interesting avenue for further exploration. (Blatt, 1983, p.57)

About adding new processes Standish writes in 1998:

> The difficulty is deciding how to choose new coefficients a_{mi}, b_{mi}, κ_i when a new process is added. There is no genotype of a process - the closest thing to it is Dawkins's meme, and there is no genetic algorithm theory of the meme. Clearly new processes arise evolutionarily, with the new processes modeled on the old. The new coefficients will be varied randomly about the old values according to some kind of central distribution. (Standish, 1998, p.77)

In his conclusion Standish writes:

> Perhaps the most important point I would like to make is that rather than studying a finite dimensional dynamical system, we should be studying what might be called open-dimensional dynamical systems, in which the number of degrees of freedom is finite, but not fixed at any point in time. ... Only then might we achieve Marshall's economic biology and have an understanding of why economic systems have evolved to be the way they are. (Standish, 1998, p.78)

In other words, we should add new columns and new rows to the von Neumann technology matrices. This gives a completely new meaning to the term "expanding" von Neumann economies. These kinds of systems are studied in artificial chemistry as we have seen, but not in economics.

There has been little experimentation with changing von Neumann matrices and the expansion is limited to input substitution (Martino and Marsili, 2005) or increased output coefficients. Vasquez's paper (Vazquez, 2003) on equilibrium growth with technological progress is an interesting example of new coefficients and the resulting new equilibrium. So far there has been no experimentation with both expanding product and technology sets, as is described in this thesis.

2.6 Toward evolutionary economic geography

As Boschma and Frenken write, economic geography should move toward an evolutionary economic geography (Boschma and Frenken, 2006). They propose an approach that was started by Veblen, and which is largely institutionalist, but that branches off when Nelson and Winter bring dynamics into the theory. This thesis argues that to truly model an evolving economy, the model needs to be a constructive system, as was discussed in the section on artificial chemistry. This is essential if we ever want to understand the constantly diversifying spatial economic system with coevolving technologies and with human actors.

In this thesis, a production system, together with the introduction and removal of products and processes, sets the stage. A newly developed free market system model provides a realistic selection mechanism. These two parts, the abstract dynamic technology set representation and the market model, capture the whole economic sphere, Kauffman's econosphere (Kauffman, 2000). It is acknowledged that we can never capture or understand human creativity and that we will never be able to predict exactly innovative use of existing products and technology. To repeat, innovation itself is unpredictable in detail: "Innovation is about surprise and surprises are not predictable" (Metcalfe, 1998, p.86). However, as has been said before, nothing can come of nothing. Innovations are combinations and new applications of existing concepts and commodities. This perspective on innovation perhaps does not allow us to predict in detail very far into the future, but it does allows to understand consequences of possible innovations that are to be expected, and it does allow us to model the consequences of more imaginative innovations. It is possible to take as a given the fact that sets of technology and products do evolve, and to model this generically. One can then learn to understand the consequences of entries and exits, and find patterns in how the system that envelopes technology is adapting to the new. The precision might not be high, but in a world of uncertainty, there is much to gain with a little increased understanding (Farmer and Lo, 1999).

This thesis develops a model which is an open ended or open dimensional system (Standish, 1998, 2008): it generates new elements and new relations. Like Schumpeter, it considers "economic evolution as an open ended process of qualitative change" (Foster and Hölzl, 2004), and like Dawid (2006) demand and innovation co-evolve. It is different from existing approaches to evolutionary economics. So far it is not the economy that evolves in evolutionary economics, but instead it is quantitative change in applied routines. Here, instead of modelling adaptive strategies, the model uses Kauffman's Lego world analogy to introduce new processes and new products. It will be shown that the model results display real evolutionary dynamics.

Chapter 3

Technology matrices and agent based modelling

The previous chapter provided an overview of evolutionary modelling in economics and economic geography. It concluded that in order to deal explicitly with an evolving system, the model needs to be constructive. This means that the model has to be capable of generating novelty during the simulation, randomly, or in response to the simulation results. The implementation of novelty, i.e. the product and technology innovation process, will be described in Chapter 4, but before this can be done, an appropriate representation of a production system has to be in place. The present chapter introduces a framework to model a simple economy. This model consists of two parts. The first part represents the fixed technology and product set by means of technology matrices. Part two deals with economic activity. In order to generate activity, we need entities capable of performing actions: agents. Activity of agents is triggered by the opportunity to generate profit. In order to define profit, a pricing mechanism is required. When these three components, the product and technology set, the economic agents, and the price mechanism are combined, a dynamic representation of a market is obtained. The final section of this chapter illustrates the individual based free-market model with a simulation.

3.1 Technology matrices

As was discussed in Section 2.4, an artificial chemistry can be represented by a triple (S, R, A), where S is the set of all possible molecules, R is a set of collision rules, and A is an algorithm describing the domain and how the rules are applied to the molecules. This chapter will apply this representation of an artificial chemistry to develop a framework for an artificial economy. In a free-market economy, the incentives for production are provided by the market. The algorithm should therefore represent a market in order to provide the rules for what is produced, when, and where. In addition, S — the set of all products — and R — the set of all production processes — need to be defined. A very efficient way of defining both sets at the same time is available from economics: technology matrices (von Neumann, 1946). Blatt provides a very good introduction to technology matrices and the corresponding dynamic economic systems (Blatt, 1983). Here Blatt's outline of technology models is followed. However, the examples developed are different from Blatt's, and are more appropriate for what is to follow.

3.1.1 A stable production system

Instead of a set S of molecules, we define a set P of products. Suppose the product set P consists of the following goods:

$$P = \{2, 3, 5, 7, 11, 13, 130, 260, 104\}$$

Here we have chosen to represent commodities by integers: primes for raw materials, and products of integers for composites. This allows us to use the structure offered by the natural numbers, and the economy so represented can be called the $\mathbb{N}$ economy. Products literally can be decomposed into their prime factorisation to see what raw materials are involved in making the products. In this model, labour is treated explicitly as a commodity (the number 2). Furthermore, there is a special product, which serves as money, and this is the number 3. A note on the use of the ℕ economy: initially the names of commodities were simply $a + b$ when the inputs were a and b, or even $coal + iron$ when coal and iron were used to produce the good. This method became very cumbersome in large simulation, and was dropped in favour of the natural number notation. The multiplication of inputs in the form of natural numbers is much more efficient, and still allows one to trace back the ingredients by factorization. The idea of the ℕ economy is borrowed from Herriot and Sawhill (2008).

Suppose we have a small product set, as above, and five production processes per time interval, as displayed in Table 3.1.

Table 3.1: Overview of the production processes during one time interval

2×products 2					→	2×products 5
2×products 2	+	2×products 5	+	2×products 13	→	3×products 130
3×products 2	+	3×products 130			→	6×products 260
6×products 2	+	6×products 260			→	6×products 104
6×products 104					→	13×products 2

Note that there is no production of commodity 13, yet it is required in one of the production processes. This product is assumed to be a free good, such as sunlight. It can easily be verified that the production in Table 3.1 is in balance: total labour (product 2) used equals total labour generated, and the products required for the generation of this labour are also covered, so that the required input per time interval is exactly the same as the output at the end of the time interval. Based on these production activities per unit of time, it is possible to determine a price set that will allow the system to function. In order for this to be the case, a producer needs to be able to finance the inputs for the next time interval with the sale of the current production. In other words, the price of the output should be greater than or equal to the price of the input. Assuming that activity levels for the next time step will remain the same, and since there is no surplus in the system, this gives the following set of equalities. When p_i stands for the price of product i, then

$$\begin{aligned}
2p_5 &= 2p_2 \\
3p_{130} &= 2p_2 + 2p_5 + 2p_{13} \\
6p_{260} &= 3p_2 + 3p_{130} \\
6p_{104} &= 6p_2 + 6p_{260} \\
13p_2 &= 6p_{104}
\end{aligned}$$

When the price of one product (the numeraire good) is taken as a price unit, for example the price of labour $p_2 = 1$ and the free good $p_{13} = 0$, then this system of equalities has a unique solution, namely $p_2 = 1, p_5 = 1, p_{13} = 0, p_{130} = 4/3, p_{260} = 7/6, p_{104} = 13/6$.

This economic system can easily be rewritten as a Leontief technology system, as in Table 3.2. This table displays two matrices. Each of the columns corresponds to a production process, and the rows correspond to products. The left matrix displays the input matrix, and each column shows the input required for that particular transformation. Similarly, the output matrix shows what product is the result of each of the production processes. For example, the fourth column of the input matrix $\{\frac{2}{3}, 0, \frac{2}{3}, 0, 0, \frac{2}{3}, 0, 0, 0\}$ shows that to execute technique four, $\frac{2}{3}$ unit of 2, $\frac{2}{3}$ unit of 5 and $\frac{2}{3}$ unit of 13 are required. The output of this process is to be found

Table 3.2: Input and output matrices representing technology and the products involved

products	*input matrix*							*output matrix*						
2	1	1	1	$\frac{2}{3}$	$\frac{1}{2}$	1	0	0	0	0	0	0	0	1
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	$\frac{2}{3}$	0	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	0	0
13	0	0	0	$\frac{2}{3}$	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	$\frac{1}{2}$	0	0	0	0	0	1	0	0	0
260	0	0	0	0	0	1	0	0	0	0	0	1	0	0
104	0	0	0	0	0	0	$\frac{6}{13}$	0	0	0	0	0	1	0

in the fourth column of the output matrix: one unit of product 130.

In the same way the other columns represent the different technologies present in the economic system of Table 3.1. Each of the columns, with the exception of the last one, requires product 2, which plays the role of labour. The last production process (the last column) generates labour at the cost of product 104, which is therefore considered to be a consumable. Sometimes processes are very efficient and all inputs are used (for example, $2+5+13 \rightarrow 2\cdot5\cdot13 = 130$), but in other processes, not all factors in the input are used: $2 + 260 \rightarrow 2 \cdot 260/5 = 104$. Product 5 is not required to produce product 104, but happens to be part of one of the ingredients. In such cases, the remainder is simply discarded. Possibly one can experiment here with pollution or externalities, but this has so far not been done.

The Leontief technology system as described here is different from the standard Leontief input-output matrix. Leontief started with similar types of data to those in Table 3.1, namely a record of transactions between sectors over a fixed time period, usually a year. Subsequently, flows from sector to sector are presented in a single matrix form, the input-output matrix, where a quotient indicates the part of the total sector output that flows to each of the other sectors. For example, when the five production processes above are regarded as five different sectors, the matrix in Table 3.3 is obtained.

Table 3.3: Standard Leontief input-output matrix representing flows among sectors. All output of sector 1 flows to the next sector, 2 in this case, and the same happens for for sectors 2, 3 and 4. The output of sector 5 is divided over sectors 1, 2, 3 and 4 in differing amounts.

sectors	1	2	3	4	5
1	0	0	0	0	$\frac{2}{13}$
2	1	0	0	0	$\frac{2}{13}$
3	0	1	0	0	$\frac{3}{13}$
4	0	0	1	0	$\frac{6}{13}$
5	0	0	0	1	0

Based on this matrix, one can calculate what the value of the total output per sector must be in order to obtain a sustainable economy, in which the income equals the outcome for each of the sectors. If we name the matrix in Table 3.3 L, the identity matrix I, and v the vector of value per sector, then $L.v = v \Leftrightarrow (L - I).v = 0$ (Weil Jr., 1964). This system of equations can be easily solved, or equivalently, v must be the eigenvector of L corresponding to the eigenvalue 1. It follows that v is any multiple of $\{2, 4, 7, 13, 13\}$. When this vector of the value of the output per sector is compared with the product output per sector (2 products 5, 3 products 130, 6 products 260, 6 products 104, 13 products 2), we obtain the same prices for each of the products as before.

The classic Leontief method provides good insight into the distribution of sector income over a particular time interval. The technology matrices in this chapter serve a different purpose. First of all, these matrices are not aggregate monetary results: every technology is treated separately even though they possibly belong to the same sector, since they possibly produce the same product. Secondly, the system consisting of two matrices does not analyse the money flows statically, as is the case in Leontief's work, but is an important part of a model that analyses economic dynamics as time progresses. In the example above, there is no input substitution possible, which means that every product can be produced in only one way. Other models, which will be dealt with later will have multiple technologies to produce the same product. The models without input substitution resemble Leontief models, as if the data have been aggregated into distinct sectors, and therefore, have been named dynamic Leontief systems (Blatt, 1983). Models with the possibility of multiple technologies per product are called von Neumann technologies. In both cases, the matrices establish the connections between commodities, and enable simulation of economic activity over time. This chapter deals with the simpler Leontief case, and the Chapter 4 deals with the von Neumann technology matrices.

Let $z = \{z_1, z_2, z_3, z_4, z_5, z_6, z_7\}$ be the vector of activities, that is, the number of times each of the columns in Table 3.2 is executed each time step. If we assume constant activity levels, we obtain the following system of equations that represent the demand and supply of each of the goods:

$$
\begin{aligned}
z_1 + z_2 + z_3 + \frac{2}{3}z_4 + \frac{1}{2}z_5 + z_6 &= z_7 \\
\frac{2}{3}z_4 &= z_1 \\
\frac{1}{2}z_5 &= z_4 \\
z_6 &= z_5 \\
\frac{6}{13}z_7 &= z_6
\end{aligned}
$$

When we take the labour production z_7 to be equal to 13, as it was in Table 3.1, then it follows easily that $z = \{2, 0, 0, 3, 6, 6, 13\}$ is a solution to this system. This illustrates solving the matrix equation:

$$(input\ matrix).z = (output\ matrix).z$$

However, we can simplify this a little. Since money 3 is not part of the production process and 13 does not have to be produced, we can discard rows 2 and 6, and the output matrix becomes a 7 by 7 square matrix. Through reordering the columns, the output matrix B can be transformed into identity matrix I. The corresponding input matrix we name A. Then $A.z$ is the input required for this activity and $I.z = z$ is the output, and these two are equal. We are now interested in solving the following systems, where p is the price vector and z is the activity vector:

$$\begin{aligned} A \cdot z &= I \cdot z = z \\ p \cdot A &= p \cdot I = p \end{aligned}$$

The vectors satisfying these relations are equal to the right and left eigenvectors of matrix A corresponding to eigenvalue 1. It can be verified that, indeed, 1 is an eigenvalue of matrix A and it is the largest real eigenvector. The Perron-Frobenius theorem guarantees positive real corresponding eigenvectors (Berman and Plemmons, 1979). In this case, the eigenvectors are $p = \{1, 1, 1, 1, \frac{1}{3}, \frac{7}{6}, \frac{13}{6}\}$ and $z = \{3, 2, 0, 0, 3, 6, 6\}$. These vectors correspond to the above stated price and activity per time interval.

3.1.2 Surplus

The previous example illustrated how, based on the production numbers per time interval, it is possible to derive an appropriate price set and an activity vector which guarantees sustainable production through the use of Leontief technology matrices. If we change one of the output coefficients in the above matrices, we get a slightly different system (see Table 3.4). As can be seen, the output coefficient of the second to last column has doubled.

Table 3.4: Adapted Leontief input output system

products	*input matrix*							*output matrix*						
2	1	1	1	$\frac{2}{3}$	$\frac{1}{2}$	1	0	0	0	0	0	0	0	1
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	$\frac{2}{3}$	0	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	0	0
13	0	0	0	$\frac{2}{3}$	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	$\frac{1}{2}$	0	0	0	0	0	1	0	0	0
260	0	0	0	0	0	1	0	0	0	0	0	1	0	0
104	0	0	0	0	0	0	$\frac{6}{13}$	0	0	0	0	0	2	0

Again, we can transform the input and output matrices into matrices A and I, where I is the identity matrix. In addition to deleting two rows and moving the last column, we now also have to divide both second to last columns by two, in order to obtain an output coefficient equal to 1. Since everything except the higher output of one activity has stayed the same, the economic system with the same activity levels as before will still function. Now this economy will generate a surplus. Since the surplus involves a consumable good, we assume for now that all the surplus will be consumed and not reinvested in the system. The activity levels of this economy will therefore be the same as those of the system above. However, the prices in this system must change, because the equality of value of output and input is used to establish the prices. If we assume that we are dealing with a competitive system, then every industry has the same rate of return r, and profits are distributed equally. We thus have to find a solution for the following equation:

$$(p \cdot A)(1 + r) = p \cdot B \quad \Leftrightarrow \quad p \cdot (A - \frac{1}{1+r} B) = 0$$

Again, this is precisely an eigenvalue and eigenvector problem: the eigenvalue of A is equal to 0.8021, and thus the rate of return for each of the industries is $\frac{1}{1+r} = 0.8021 \Leftrightarrow 1 + r = 1.2468$. Surplus thus leads to profit rates larger than 0 and prices corresponding to this return rate are

$$p = \{1, 1, 1.24679, 1.24679, 1.24679, 0, 1.86753, 1.78761, 1.73779\}$$

3.1.3 Balanced growth

Suppose now we have matrices as in Table 3.5. These matrices represent the technology set in an economy. Each of the columns can be regarded as a transformation of products, and shows what can be obtained by using certain combinations of goods. Again let A be the input matrix and B the output matrix of which, in both cases, rows two and six have been removed (money and the free product). The role of the money element 3 will become clear when the market model is explained.

As the previous example shows, it is possible to generate surplus in such systems when output is larger than the input required in the next time interval. Above, the surplus was destined for discretionary consumption, but what happens when all the surplus is reinvested in the system? A first possibility is that the system displays

Table 3.5: Leontief technology system

products	*input matrix*							*output matrix*						
2	1	1	1	1	1	1	0	0	0	0	0	0	0	6
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	0	0
13	0	0	0	1	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	1	0	0	0	0	0	4	0	0	0
260	0	0	0	0	0	1	0	0	0	0	0	2	0	0
104	0	0	0	0	0	0	1	0	0	0	0	0	$\frac{7}{3}$	0

balanced growth. This means that the activity of every sector grows exponentially, with constant factor a, $z(t+1) = az(t)$. For the equations regarding the activity levels, this implies

$$A \cdot az = B \cdot z \Leftrightarrow (A - \frac{1}{a}B) \cdot z = 0$$

The solution to this equation is the generalized right eigenvector corresponding to the largest real eigenvalue and it is easily verified that $a = 2$ and $z = \{6, 0, 0, 3, 6, 6, 7\}$. For example, the output of such z is $B \cdot z = \{42, 6, 0, 0, 12, 12, 14\}$. The input required for this level of activity is $A \cdot z = \{21, 3, 0, 0, 6, 6, 7\}$. Indeed, the output is twice the size of the input, and when all surplus is reinvested in the system, this allows the economy to grow exponentially: $z(t+1) = az(t)$. This is the famous von Neumann balanced growth situation, where the system is stable and expanding at the same time (quasi-stationary equilibrium (Champernowne, 1946)).

The return rate and price vector are determined likewise:

$$p(1+r) \cdot A = p \cdot B \quad \Leftrightarrow \quad p \cdot (A - \frac{1}{1+r}B) = 0$$

The largest real eigenvalue of A with respect to B is 2, the return factor $1 + r$ is equal to 2, and therefore, the return rate r is equal to 1, and this eigenvalue corresponds to the left eigenvector $\{1, 2, 2, 2, \frac{3}{2}, \frac{5}{2}, 3\}$ when labour is chosen as the numeraire good. A return rate of 1 and a growth factor of 2 are perhaps not very realistic, but the system serves perfectly well as an example.

Another possibility of what to do with surplus has already been mentioned: the surplus can wholly or partly be consumed. The remainder is stored. In this case, the return rate, and thus return factor, remains the same: prices are based on production numbers and not on the final destination of the products. The growth factor, however, decreases with consumption, as there is not enough production to secure the maximum growth rate (2 in the example above). Discretionary consumption can sustainably be at levels between 0 and the total surplus, and higher consumption slows down the growth of the system. According to Blatt, this is in agreement with Adam Smith's remark "Every prodigal appears to be a public enemy, and every frugal man a public benefactor" (Blatt, 1983, p.102). However, often consumption is seen as a driving force as well, and this is the case in the individual or agent based model that will be introduced below.

This thesis assumes a priori no growth in activity levels: all surplus is either consumed, or it is stored, and consumed or used for production later. Expansion of the system is left to the system itself, either by increased discretionary consumption levels or by an increased number of consumers. However, the equilibrium price set does not depend on consumption levels and can always be calculated, and with a known level of discretionary consumption, the activity levels can be calculated.

$$(A - B) \cdot z - consumption = 0 \Leftrightarrow z = (A - B)^{-1} \cdot consumption$$

We will see that in Leontief economies, such as explained above the agents introduced in the next section are well behaved, and that their aggregate behaviour corresponds to the activity levels calculated by the inverse Leontief matrix.

3.2 The individual based model of a market

The previous section described a method to represent the product and technology set. To determine what rule is applied when and where, the artificial chemistry framework requires an algorithm. This algorithm can have many different forms; for example, Kauffman lets the king of France make all decisions (Kauffman, 1995). Another possibility is to minimize activity levels while maximizing output. These methods require an all-knowing ruler who can decide what is best. Although some putative examples of economies that are ruled in such a way exist, a more practical approved interpretation is to let individuals decide what is best according to their knowledge. The algorithm developed here is such a case. It resembles a free-market, which means that individual economic agents, representing firms and consumers aim for the most profitable state they can obtain. This section explains the implementation of the interacting agents and the price mechanism used to guide all actions, for a more concise description see Section B.2 in Appendix B.

3.2.1 Space

At the cost of labour, raw materials are extracted from the land, which has been divided into cells of equal size. As far as raw materials are concerned, the space is homogeneous. If desired, it is possible to experiment with more interesting distributions of resources.

Each cell provides a free resource, 13, which can be thought of as energy from sunlight. The distribution of free energy can be varied, but currently it is such that there is always an abundance.

On top of the land is a connection network, which represents the presence of trade links between cells. Links go both ways, so if a cell a is connected to a cell b, then b is connected to a, and all agents on a can trade with all agents on b and vice versa. Initially the network is empty, but as agents obtain skills and require inputs for these skills, they can establish connections with providers. A wide range of rules of how to expand or contract an agent's trade network is possible. For example, an agent can randomly select one of its current providers and subsequently do a local search around the selected trade partner in an attempt to find an additional provider. Or an agent may be allowed to connect to the nearest provider. In addition, it is possible that an agent looses one of the more distant connections and subsequently attempts to find a supplier nearer by. Such rules obviously attempt to keep the social network consisting of trading partners compact. The rules according to which agents behave are described in the next section.

3.2.2 Agents

The grid space is home to a number of economic agents. They all have a fixed location and an identity number, so that we can distinguish different agents on one location (cell). Furthermore, agents possess assets: resources, other products present in the economy and skills. All assets are stored in a list. Therefore, an agent can be represented by

$$\{\{x, y, z\}, \{p_1, \ldots, p_n\}, \{t_1, \ldots, t_m\}\}$$

where x, y is the location, z is the identity number for that agent. $p_1, \ldots, p_n$ lists the possession of n products, and $t_1, \ldots, t_m$ is a boolean list to specify which of the m technologies the agent possesses for some $m, n \in \mathbb{N}$. Different agents have different skills.

Having a technology should be regarded as having the skill or knowledge to perform a certain transformation when you have the resources to do so. For actions that require capital, in addition to having the skill, the agent must also possess the appropriate capital goods in order to execute that particular action. Capital goods are a subset of the product list P.

As mentioned before, some products are consumables, while others are capital or intermediate products. The consumables can be converted into labour, 2, according to certain columns in the technology matrices. The actions of these particular columns represent consumption, and consumption is restricted to a certain type of agent, namely the group of consumers. We thus distinguish two types of agents: consumers and producers. The latter can have any of the other technologies. Producers depend on consumers for the required labour, while consumers depend on producers for the consumables. Neither group can have skills that belong to the other group: an agent is either consumer, or producer, but never both. This is not crucial to the functioning of the model and could easily be relaxed later.

Figure 3.1 shows a grid of size 14 by 14, and agents are scattered randomly over the cells. Some cells have no agents, while others contain an agent with a random selection of non-consumption skills. In order to give each producer access to labour, each location with an agent obtains a second agent with only consumption skills. The map in Figure 3.1 shows 38 locations with agents, two agents on each of the populated cells: one producer, and one consumer.

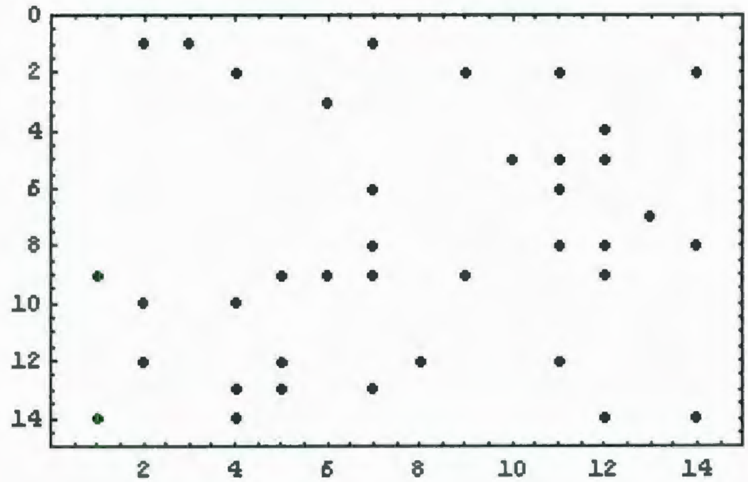


Figure 3.1: The agents in space

3.2.3 Agents in action

The production of goods, mentioned in the previous section, does not simply happen by itself. It occurs because somebody (something) somewhere actually does the job. This follows the approach of the increasingly popular individual or agent based modelling. Unlike Jain and Krishna (1999), there is not an assumed population behavior at the macro level, but one that is generated by individuals themselves. Macro scale properties are derived bottom up. Therefore, we need actual agents in space to serve as economic individuals to drive the economy. The whole environment of space, networks, agents, product set and technology is called the economy (econosphere in Kauffman (1995, 2000)). All changes that take place in the economy occur because agents perform some or all of the following actions:

Win free resources: Free resources are are distributed over the cells.

Expand network: An agent now asks around among its trading partners (all agents it is linked with) to inquire what they have in store. It compares this list with what is possibly required for executing its own skills. If there are any products it does not have access to, it will randomly choose a trading partner, then scan the surroundings of the trading partner for interesting new partners. A connection to the cell that contains the agent that offers most of what was not yet available will be added to the connection network.

Make plan: What an agent can do, firstly depends on its skills. Secondly, it needs to have the resources to do it. Resources can be bought. However, the agent is limited by what partners have to offer. A surplus of products can be sold to generate money to buy the required resources. However, the quantity of products being sold is dependent on the monetary resources of trading partners. In the end, the agent's initial possession, plus what is acquired, minus what is sold, minus what is used in the production process, plus what is produced must be positive. Given these constraints, and given the prices of all commodities (determined by an auctioneer explained below), every agent uses linear optimization to determine what and how much to sell, what and how much to buy, and what and how much to produce in order to achieve maximum possession.

Buy: Once an agent has optimized its plan, it will look for a partner that has to offer what it is looking for, and they will exchange q units of product l for $q \times p_l$ units of money, where p_l is the current value of product l.

Sell: Once an agent has optimized its plan, it will look for a partner that has the money to buy what the agent wants to sell, and they will exchange q units of product k for $q \times p_k$ units of money, where p_k is the current value of product k.

Produce: After the exchange of products and money, the agent has all the resources required for the production plan and it can transform the ingredients into its output.

Update: An agent executes all of these steps, and the state of the agent and the agents engaged in trade are updated.

Unlike cells in cellular automata, the agents cannot be updated synchronously, because one agent's action will change the state of its partners with whom it engages in trade. Therefore, the agents perform sequentially. All the agents are randomly ordered, and in turn they go through the above list of actions. When everybody has had a turn, a new randomly ordered list is created for the next iteration. A complete sequence of agents' actions defines one iteration. The random list of agents generated anew for each iteration prevents any bias due to specific ordering of the agents. The next section discusses the agent's decision mechanism in more detail.

Linear programming to maximize profit

Suppose the input matrix and output matrix are A and B. Dimensions of these matrices are $n \times m$, meaning that there are m production processes and that the economy consists of n products, with $m, n \in \mathbb{N}$. As in previous examples, the first product is labour and the second product represents one unit of money. Now expand these matrices by adding the matrices M and I:

$$
\begin{aligned}
i &= \left(A \mid M \mid I \right) \\
o &= \left(B \mid I \mid M \right)
\end{aligned}
$$

where I is the identity matrix of size n, and M is the square matrix of size n with all elements equal to 0 except for the second row which is equal to $p = \{p_1, p_2, \dots p_n\}$. For example, in an economy with $n = 3$ products $I = \{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$ and $M = \{\{0, 0, 0\}, \{p_1, p_2, p_3\}, \{0, 0, 0\}\}$. The addition of these two matrices to A and B represent all possible actions involving production, buying and selling. Just as columns in A and B represent the input and output for production processes, the columns in M and I represent input and output for buying products. The input is the price of a product, the output is the product itself. Selling simply does the opposite. When we write $z = \{z_1, z_2, \dots z_{m+2n}\}$ as the vector of all actions of an agent, that is, z consists of elements $z_1, \dots z_m$ to indicate the activities regarding the m production processes, $z_{m+1}, \dots, z_{m+n+1}$ to indicate the quantities of products that need to be bought, and $z_{m+n+2}, \dots, z_{m+2n}$ the quantities of products that need to be sold, then $i \cdot z$ lists the quantities of products required for the execution of vector z and and $o \cdot z$ lists the quantities of products generated by vector z. The quantities $i \cdot z$ and $o \cdot z$ are named the input and output, respectively, of the vector of action z.

Likewise, when the vector $p = \{p_1, p_2, \dots p_n\}$ lists the prices for each of the products, then $p \cdot (o - i)$ is a vector that gives the profit of each of the actions. When activity levels and profit per activity are combined, we obtain the following expression for the profit:

$$profit(z) = p \cdot (o \cdot z - i \cdot z) = p \cdot (o - i) \cdot z$$

The agent will maximize profit, but in doing so, it has several constraints. Most importantly, the final balance of products has to be positive. If the assets of an agent, the set of products that the agent has at the beginning of the turn, is a −

$\{a_1, a_2, \ldots a_n\}$ then the end result of its actions should be positive for each of the products.

$$a + (o - i) \cdot z \geq 0$$

Buying and selling options are naturally constrained: you cannot sell more than you have or than your trade partner can afford, and you cannot buy more than what is offered. Trading for a product is limited to one transaction with one agent per product per turn. Therefore, the maximum quantity of a product that an agent can buy is determined by the agent that, of all trading partners, possesses the most of that particular commodity. This means that per product, an agent can have only one supplier, although next turn, of course, it can have a different supplier for that same product. Within one turn, different products can be bought from different sellers. The fact that, during one turn, an agent cannot buy a product from more than one seller is not realistic, but it makes programming much simpler. Chapter 7 will provide an alternative. The constraints that apply to buying are as follows:

$$\{b_1, \ldots, b_n\} \leq \{\max_{tp} c_1(tp), \max_{tp} c_2(tp), \ldots \max_{tp} c_n(tp)\}$$

where tp goes through the list of trading partners and $c_i(tp)$ equals the stock held by trading partner tp of product i for some $i \in \{1, \ldots, n\}$.

Similarly, the maximum quantity that an agent can sell is determined by vector a. So if we write $z = \{z_1, z_2, \ldots z_{m+2n}\} = \{m_1, \ldots m_m, b_1, \ldots b_n, s_1, \ldots s_n\}$ to break up the activity level vector into the different parts of manufacturing ($m_.$), buying ($b_.$) and selling ($s_.$), then

$$\{s_1, \ldots, s_l\} \leq a$$

There is a time constraint; an agent is limited in how much it can do in one turn.

$$\sum_{i=1}^{m} z_i \leq maxproductivity$$

Note that buying and selling are excluded, the summation only adds the activity levels regarding production. Apart from the total activity being limited by a parameter *maxproductivity*, the agents actions are also constrained by its technology set. Remember that $\{t_1, \ldots t_m\}$ is the technology list of an agent, which is a boolean vector with 1s and 0s for skills it can and cannot perform.

$$\{m_1, \ldots m_m\} \leq maxproductivity \cdot \{t_1, \ldots t_m\}$$

To complete the list of constraints, one more is mentioned here. Capital has not been involved in the production process so far, and the implementation of capital will be explained in Chapter 4. However, when capital is involved, the quantities of transformations that require capital are furthermore limited by the quantity of capital the agent possesses. If $c_1, c_2, \ldots, c_k$ are the capital goods required for executing techniques $t_{c_1}, t_{c_2}, \ldots t_{c_k}$ then

$$\{z_{c_1}, z_{c_2}, \ldots z_{c_k}\} \leq \{c_1, c_2, \ldots, c_k\}$$

which implies that you cannot execute a technique more often than what you have capital for.

Ultimately, all demand originates from the consumers' desire to acquire consumption goods, "production follows needs" (Schumpeter, 1961, p.12). In the model consumption results in the capacity to perform work, (it results in a product called labour), which is required as input in the execution of all manufacturing processes. No demand for consumables and no manufacturing leads to the absence of demand for labour and thus no demand for consumables: the stable 0 state. To prevent this trivial state from occurring, consumers are encouraged to spend their income on consumables (which will allow them to perform work), even though there is no immediate demand for the labour that results from these consumables. There are several ways this can be achieved, one possibility is explained here, another possibility will be introduced in Chapter 5.

Here, the model has an exogenously determined consumption parameter (c), which makes the consumers among the agents consume for the sake of consumption. Consumers will certainly generate labour (i.e. consume) when there is excess demand for labour and thus generating labour is profitable. In the case that labour supply is sufficient, consumers will still consume based on this parameter. The parameter c tells the consumer group among the agents how much consumption has to take place each turn, regardless of the situation on the labour market. c does not specify which goods need to be consumed, but indicates a quantity of consumables equivalent to c units of labour. When the parameter c is set at 0 we have a perfect thrift system, since all consumption results in labour, whereas when the consumption is maximal, such that all agents are forced to use their maximum productivity level, all surplus is consumed for the sake of consumption. In between these two parameter values, part of the surplus is consumed, and part is stored for later use. Whatever the situation may be, for every consumer the expenditures on consumables of a consumer $z_{l_1}, z_{l_2}, \ldots z_{l_n}$ have to generate more than c units of labour, and these consumables are removed from the system without resulting in labour.

$$\{z_{l_1}, z_{l_2}, \ldots z_{l_n}\} \cdot \{o_{l_1}, o_{l_2}, \ldots o_{l_n}\} \geq c$$

This consumption parameter c effects only the consumer group. Producers, which can only produce, and depend on the consumer group for their labour requirements, are free from consumption. However, in their case it is possible to implement wear and tear by reducing the capital by a certain factor each time it is involved in a production process. This provides the producers with depreciation costs. Alternatively, one can have aging capital as in von Neumann (1946).

The behaviour of the economic agents is

$$\max_{z \in \mathbb{Q}^{m+2n}} profit(z) = \max_{z \in \mathbb{Q}^{m+2n}} p \cdot (o - i) \cdot z$$

under the constraints as listed above. Although each of the agents uses the same behaviour rules, the behaviour will be very different for different agents. They possess different skills, assets, and trading partners. Furthermore, the price vector p varies over time as a result of mismatches between supply and demand. The next section explains the working of the price mechanism.

3.2.4 An incentive for action

An agent acts in its local environment. In other words, an agent's environment consists of itself and its trading partners. The agent does not perceive how the whole system is doing. It just acts within the boundaries set by its trading connections. In order to obtain a functioning economy, where resources are scarce and have to be used in an efficient way, something more than just local interaction is required. On the other hand, we do not want to prescribe the agents' actions, as this would resemble having a social planner for the whole economy (Kauffman, 1995). This model uses an individual based market model, and resembles Adam Smith's invisible hand to steer the economy through times of scarcity. This is achieved by means of a price mechanism.

In the western economic system, the market drives the economic activity. There is a large volume of literature on price formation. However, in most cases the resulting prices are equilibrium prices (one of these theories of price formation was used above to determine equilibrium prices for the technology matrices). There also exists a relatively new body of research that deals with agent based models of price negotiations in an attempt to be able to predict stock market price dynamics (Tesfatsion and Judd, 2007). These models focus on the agent's price setting tactics or strategies, and the main focus is to determine the best strategy or adaptive strategy. This thesis focuses on the functioning of a whole economy, and the price itself is important only in order to create a functioning market operation. Instead of following fairly complicated models of price strategies, here we have developed a much simpler model to define prices and to balance supply and demand, and this with good result.

The price mechanism

When the agents go through their production and trading actions, the model keeps track of what is required during the last w iterations (often $w = 50$). Subsequently, the agents attempt to keep in store the required input for those w iterations, history governs the activity of the individual (Schumpeter, 1961, p.6).

Let ea be an economic agent, and $n_i(ea)$ the stock of product i that agent ea possesses. When the n_i are summed over all agents, we obtain $inventory = \{\sum_{ea} n_1(ea), \sum_{ea} n_2(ea), \ldots \sum_{ea} n_n(ea)\}$. When the global demand of the last w iterations is $consumption + input = \{d_1, d_2, \ldots, d_n\}$, then the price mechanism looks at the sign (positive or negative) of each of the elements of

$$inventory - demand = \{\sum_{ea} n_1(ea) - d_1, \sum_{ea} n_2(ea) - d_2, \ldots \sum_{ea} n_n(ea) - d_n\}$$

The price mechanism has an auctioneer who attempts to find a set of prices such that, for those products for which the above equation denotes a shortage, the production process becomes profitable (cost of input lower than price of output), and the production processes of products for which there is a surplus, manufacture at a loss (cost of input higher than price of output). The prices are similar to Sraffa's market clearing prices based on the cost of input. When dealing with a shortage, the value of the output is such that it enables the sector to buy the required input and produce with a profit. Through this, agents are encouraged to reduce shortages and surpluses.

Local prices

One disadvantage of the above price mechanism is that the prices are determined globally. A shortage in one location is balanced out by a surplus in another location, even when there is no trade between these two. As a result, a local shortage is possibly not noticed by the auctioneer, and prices stay as if the system were in balance. This problem can be remedied by making the price mechanism local. Before an agent (call this agent temporarily the active agent) decides what to do, it asks the auctioneer for the current prices. As before, the auctioneer determines the demand $\{d_1, d_2, \ldots, d_l\}, d \in \mathbb{N}$ in the last w iterations, but this time the auctioneer does so *locally*. The local market is determined by the set of agents connected to the active agent by its trade network. Then a comparison of what has been demanded and what is locally in stock, as before, gives *local* shortages and surpluses, and again prices are set. Based on these prices, the currently active agent maximizes the outcome of its actions: the profitable actions will be executed as much as possible, and the ones that result in a loss, as little as possible. The algorithms for the model including situated agents and local pricing can be found in Appendix B.3.

The price mechanism is perfectly capable of reducing local shortages and surpluses, and at the same time, the global market, which is composed of many different overlapping local markets, is kept in balance. The next section will provide evidence for this claim.

3.3 A fixed model run

3.3.1 Setup

With all the above in place, we can run the model to simulate production and trading processes. Suppose we have the following set of products: labour and money, raw materials and composites.

$$
\begin{aligned}
\texttt{labour} &= 2 \\
\texttt{money} &= 3 \\
\texttt{raw materials} &= \{5, 7, 11\} \\
\texttt{free resource} &= 13 \\
\texttt{composites} &= \{130, 260, 104\} \\
P &= \{2, 3, 5, 7, 11, 13, 130, 260, 104\}
\end{aligned}
$$

The input and output matrices in Table 3.6 denote the technology set. Note that these matrices are not square, but by deleting the rows 2 and 6 which involve

Table 3.6: The possible transformations, where three of the output coefficients are set to 2 and the final output coefficient is determined such that all sectors have an equal profit rate of 5%.

products	*input matrix*							*output matrix*						
2	1	1	1	1	1	1	0	0	0	0	0	0	0	$\frac{29491521}{25600000}$
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	0	0
13	0	0	0	1	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	1	0	0	0	0	0	2	0	0	0
260	0	0	0	0	0	1	0	0	0	0	0	2	0	0
104	0	0	0	0	0	0	1	0	0	0	0	0	2	0

the products money 3 and the free product 13 we can obtain 7×7 matrices which still contain all the relevant products. This allows us to treat the price and activity vectors as solutions to an eigenvector problem. Let A and B denote the squared input and output matrices.

The largest real eigenvalue of A with respect to B is $\frac{20}{21}$. This means that a price vector (the corresponding eigenvector) exists such that each of the processes results in a 5% return on the value of the input.

$$p \cdot (A - \frac{20}{21} B) = 0$$

It can be verified that $p = \{1, \frac{21}{20}, \frac{21}{20}, \frac{21}{20}, \frac{861}{800}, \frac{34881}{32000}, \frac{1404501}{1280000}\}$ or any multiple of p is a solution of the above equation. By inserting price 1 for money and price 0 for the free product, an equilibrium price set for the above input and output matrices is obtained. During simulations, products will be locally priced around this equilibrium price: prices will locally be higher when shortages appear and lower or equal to the equilibrium price when supply is sufficient. In general, each of the profit rates will be different, and moreover, profit rates per technology will differ from the standard 5% rate. The actual pricing and corresponging profit rate depend on the local balance between supply and demand for each of the products.

As for the intensity levels, we need to find a solution for z in the following equation

$$(B - A) \cdot z = \{consumption\} \Leftrightarrow z = (B - A)^{-1}\{consumption\}$$

where *consumption* is the total quantity of consumables that each iteration are consumed because of the exogenously set consumption parameter c. This equation is similar to the Leontief inverse matrix technique to calculate activity levels for a certain external demand. Here, external demand is replaced by consumption, and the z of the above equation gives the activity levels required to generate enough consumables both for labour and for consumption. Note also that the eigenvalue is missing in the equation. Growth is not assumed to be constant. However, growth is possible, and it will depend on the number of agents and their activity in the market, or on increasing consumption.

Suppose now that each of the consumers provides labour when there is demand for labour through normal consumption. In addition to this, each consumer is forced to consume at a level of $c = \frac{3}{10}$. This implies that at each turn a consumer has to take in an amount of consumables equivalent to $\frac{3}{10}$ units of labour. These consumables are not transfered into labour, but are simply taken out of the system (consumption for the sake of consumption). Or what is effectively the same, these consumables are transfered into labour, and that labour is taken out of the system. In either case the consumables disappear. The method of expressing the forced consumption c in units of labour is preferred because in the case there are more consumables, as there will be later, every agent is free to choose from all consumables, as long as the consumables are equivalent to c units of labour. The total forced consumption then is $\frac{3}{10} \times \|cons\|$, where $\|cons\|$ denotes the number of consumers. In Figure 3.1, there were 38 producers and an equal number of consumer scattered over the map. Total consumption is thus $38 \times \frac{3}{10} = \frac{57}{5}$ This gives the following *consumption* vector $\{\frac{57}{5}, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$. It can be verified that $z = (B - A)^{-1}\{\frac{57}{5}, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \approx \{9.37, 0, 0, 9.37, 18.7, 37.5, 74.9\}$.

3.3.2 Leontief results

Suppose we start a simulation with agents on a grid as in Figure 3.1, and with an economy as given in Table 3.6. The corresponding flow of commodities of this product and technology set is displayed in the directed graph in Figure 3.2. The products are ordered counterclockwise, and the arrows indicate which products are required for the production of each of the goods. It shows that `2` (labour) is required for each of the processes, and that `2,5,13` result in product `130` and so on. The production processes are similar to those in Table 3.1.

We provide every agent with a random list of technological skills, and with a fixed list of assets $\{1, 100, 6, 4, 4, 2, 1, 1, 1\}$ to allow them to start. Remember that an agent is represented by a list of coordinates, assets and technologies. The agent in the top left corner of Figure 3.1 is, for example, given by

$$\{\{1, 2, 1\}, \{1, 100, 6, 4, 4, 2, 1, 1, 1\}, \{1, 1, 1, 0, 0, 0, 0\}\}$$

Since the production history is unknown, it is assumed that it is equal to 0 for all $u = 50$ iterations. An iteration is a series of actions where each agent gets a turn to perform the various actions. Every iteration contains all agents, but for every

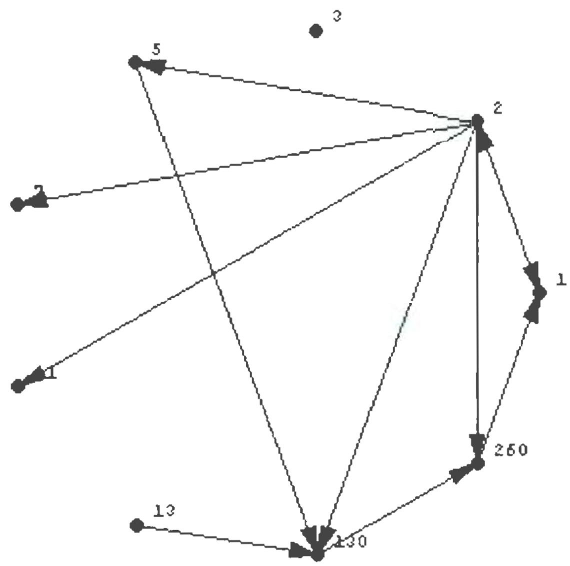


Figure 3.2: The flow of commodities through the economy

iteration, the agents are ordered randomly. Initially the local stocks are sufficient because current inventory is larger than the previous demand. Because the price mechanism assesses the difference between what is locally in stock and what has been demanded in previous iterations, there is no drive for production based on the initial configuration. If, in addition, the consumption parameter was equal to 0, then the model would be in a trivial stable state, as none of the agents would see any reason to start production. However, with consumption c larger than 0 ($\frac{3}{10}$ in this case), the consumer group begins to demand consumables, and depletes the stock. As soon as their consumption becomes larger than what is in stock, a shortage is detected and prices begin to change, making it profitable to start production of 104, the consumable in this case. Initially the input for this activity comes from whatever is in store, but soon these stocks will be depleted too. After a number of iterations, the whole production process has been started. If we continue to run this system for about 500 iterations, the production numbers displayed in Figure 3.3 are obtained.

The left column of Figure 3.3 shows the raw production numbers per iteration for each of the products involved in the cycle. As mentioned earlier, the production history at the start of the system is empty, and this is represented by 0 level production activities. As time progresses, the stocks are depleted, and the production cycle, as in Figure 3.2, is developed. The production per iteration differs from the previous and the next in a quite erratic manner due to the rather small number of agents in this run. One agent makes up a large part of the aggregate behaviour. Ultimately, all products are constantly produced, sometimes leading to an overproduction after which production is lower, until stocks are depleted again. This is an inventory cycle showing a cyclical pattern of over and underproduction. When the price window parameter, which is used to set the length of the memory of the system, is longer than the period of the inventory cycle, the system is dampened as is illustrated in the figure.

The right column of Figure 3.3 shows the same production data, but here the numbers are averaged over a gliding window to filter out the high frequency cyclical over and underproduction. After the start of the simulation, the gliding average production per iteration fairly quickly stabilises at approximate levels $\{86, 9, 19, 37, 75\}$. Very interesting is that these average production numbers correspond precisely to the output of the activity vector z calculated by means of the inverse Leontief matrix.

$$B \cdot z \approx B \cdot \{9.37, 0, 0, 9.37, 18.7, 37.5, 74.9\} = \{86.3, 9.37, 0, 0, 18.734, 37.468, 74.936\}$$

This illustrates that the local market and its price mechanism function very

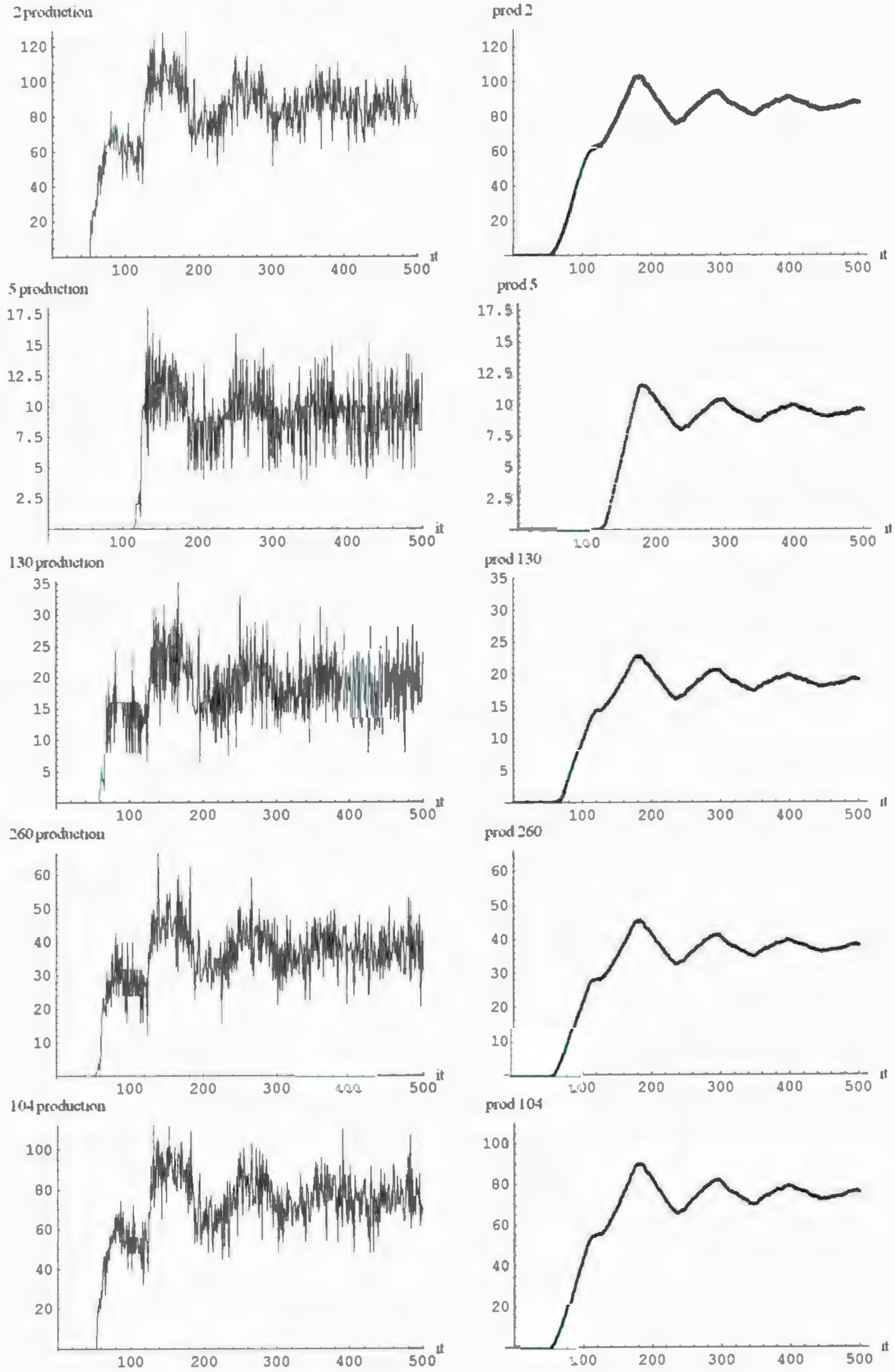


Figure 3.3: Production of five products during a simulation run. The left column shows the raw production numbers per iteration, while the right column displays the same data but now as a gliding average.

well. It also illustrates Adam Smith's invisible hand that guides the economy. Local agents are only aware of shortages and surpluses in their direct neighbourhood through the contacts they have with their trading partners. This results in local prices, based on which, as Adam Smith proclaimed, each individual will strive for maximal personal gain. When all agents are doing so, the average aggregate behaviour corresponds precisely with the required activity levels calculated by means of the Leontief inverse technique. Agents do overproduce or underproduce, but only temporarily. A well functioning market with competing economic agents is thus capable of guiding economic activity, and results in an appropriate global economic behaviour. Adam Smith mentioned this over two centuries ago, and to my knowledge, this is the first model capable of actually simulating this process. This is not the main goal of this thesis, but it does show that the market mechanism, which will be used here to select successful and unsuccessful innovations, is functioning properly.

Chapter 4

Innovation, entry and exit

Manipulating the input or output coefficients in technology matrices, as happened in an example in the previous chapter, is not uncommon in economics. Indeed, this is precisely how innovation is often treated in evolutionary economics (Nelson and Winter, 1982; Dosi *et al.*, 1995). However, the actual underlying change in process is never specified. A very short section in Morgenstern and Thompson (1976) is an exception, but it is merely a sketch, and is not explored in any detail. Furthermore, simply changing coefficients does not include new products, or create new connections, and these are essential ingredients of innovation. This leaves innovation as the outcome of a black box (Dawid, 2006), even though it has been recognized as a primary source of industrial dynamics (Dosi *et al.*, 1997). Innovation clearly deserves, and requires, more attention.

The addition of a truly new production technique to the system, or the introduction of a new product into the market, this thesis will show, is a much more appropriate way to include innovation in an economic model; doing so means that the structure of the model itself evolves as the model is run. A new production technique represents a completely new function, and a new relation among the inputs. All of the products involved may have already been in the system, but, until then, it was never realized that they had such a use. As a result, innovation is not a random change of one of the input or output coefficients, such as the labour and capital input coefficients a_l and a_k in Nelson and Winter (1982). Innovation is a completely new application of existing stock. The line of thought developed in this chapter is to expand the technology matrices of the previous chapter to a von Neumann technology set, and subsequently, to expand the matrices repetitively so that they represent an open dimensional dynamical system that includes innovation and diversification. New columns represent new relations and new functions, i.e. new technology. Some innovations will require the introduction of new products to the system. In a similar fashion, new products can be added to the system by expanding the number of rows. Adding columns and rows to technology matrices will provide a much more comprehensive method of dealing with innovation.

4.1 Innovation

The principle used to simulate innovation is based on Kauffman's Lego world, or constructive systems in artificial chemistry. New products appear by joining already existing products. Products can function as capital, intermediate products or consumables, and thus we will see new forms of such functions. Each addition of a column to the technology matrices implements new technology. This, in a nut shell, is how innovation is dealt with in this thesis.

Many cases of innovation throughout history can be approached this way. Rosenberg acknowledges the path-dependence of technological change, and the dependence of new technology on the old. "Present activities are powerfully shaped by technological knowledge from the past" (Rosenberg, 1994, p.14). Here this line of thought is extended: if current technology is based on technology of the past, then future technology is shaped by current technology.

Having said that, it is clear from the outset that innovation is a difficult concept that is not easily captured by rules. We agree with Metcalfe that innovation itself is unpredictable in detail: "innovation is about surprise and surprises are not predictable" (Metcalfe, 1998, p.86). Subsequently, Metcalfe argues that one has to focus on differences between companies in their quest for innovation, not on the inventions themselves.

> Thus we must imagine our selection set as a continually changing set of product and process combinations, and those innovations which are fundamental in an evolutionary sense will be those which change its boundary. (Metcalfe, 1998, p.87)

This leads to the approach taken by Nelson and Winter, and the evolutionary economists. However, as has been said before, nothing can come of nothing. Innovations are combinations and new applications of existing technologies and commodities. This perspective on innovation may not allow us to predict in detail, but it does enable us to model the consequences of artificial innovations. This approach allows us to deal explicitly with what Metcalfe suggests we have to imagine: the continually changing product and process set.

Burke (1978) gives a wonderful, though popular, overview of the links between different technologies, and it provides much support for the approach advocated by this thesis. The book consists of examples of connections between already existing techniques or products that then result in new techniques or products. Often these connections appear quite random. Initially, the techniques that will become connected appear to have nothing in common. A striking example of this phenomenon is an invention by Edison, who combined contemporary ideas as if part of a jigsaw puzzle: the light bulb, the billiard ball, the zoopraxiscope and the phonograph. These four seemingly random pieces, or parts thereof, were fit together to create the kinetoscope. The billiard ball of those days was no longer produced from ivory due to short supplies. Instead the shell was made out of a newly developed material, celluloid. The zoopraxiscope was capable of giving the illusion of motion out of static images. The light bulb and phonograph are still well known. The combination of these products or techniques to create the kinetoscope seems almost random, yet its application a century later in cinemas is still based on the same principle. This illustrates two characteristics of innovations: the novelty generated by seemingly random combinations, and, at the same time, the unpredictability of these combinations.

4.1.1 An example

The present work does not pretend to be capable of predicting innovation: this is clearly out of reach. However, it does mirror the construction of innovation, by somewhat randomly constructing new elements out of existing ones. This allows the study of innovation and its consequences for the production system. Table 4.1 illustrates how realistically the innovation process can be captured by the method proposed here. This table displays part of a chapter on the development of energy sources from *Connections* (Burke, 1978). Instead of describing the evolution of power generation technology, as Burke has done, these two matrices show the evolution in von Neumann technology matrix form. The continual expansion of the matrices over time, visualized by the different shades of grey, appears like layers of an onion, encapsulating the expanding state of the art. The following paragraphs briefly summarize Burke's chapter and describe the developments in the von Neumann matrices of Table 4.1. Both the table and the summary give a simplified account of the developments, but they suffice to illustrate the use of expanding von Neumann technology matrices to model the innovation process.

Around 1500 A.D. window glass became a much more widely used consumer product, whereas previously it had been used almost exclusively for religious buildings. The first columns `a` of the input and output matrices show that labour, sand, potash, lime and wood are used to produce glass. When glass gained popularity, better ovens were developed for producing glass, but wood remained the source of energy fueling the ovens (columns `b`). The glass industry had to compete with the manufacture of iron (columns `c,d`), which was used among other things for casting iron cannons from 1543 onwards (columns `f`), and ship building (columns `e`) for the dwindling wood reserves in England and, to a lesser extent, Europe. Due to the very serious shortage of wood around 1600, the glass industry was in a crisis since priority was given to the other two industries. At that point, coal was already known (columns `g`) but used on a small scale due to the impurities coal fires caused in the finished product, whether glass or iron. In 1611, a new method of glass making was developed, which, through the use of a different furnace, was able to make glass by using coal (columns `h,i`). With the new furnace, coal could be used in the making of glass, as well as in the process of making metals, such as brass and iron (columns `j, k`).

Increased industrial activity required many more raw materials, such as coal, copper, zinc and iron ore. With the increasing need for transportation to ship these

Table 4.1: Burke's development of industrial power captured in technology matrices. The development of power generating technology is illustrated by expanding von Neumann technology input and output matrices. Expansions upwards in the tables illustrate the use of a new raw material, expansions downward show the introduction of a new composite product. The expansions to the right illustrate the application of the product set. The dates on the left indicate the year of introduction of the new technology.

Input matrix

		a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
1611	brick	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
	coal	0	0	0	0	0	0	0	0	1	1	1	0	0	1	1	1	1	1	1	1	1
	zinc	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	copper	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
1543	iron ore	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
1500	labour	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	sand	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	potash	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	lime	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	wood	1	1	1	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1500	glass	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	oven	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	charcoal	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1543	iron	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
	ships	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	canon	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
1611	furnace	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
	brass	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1690	Savery	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0
1704	Newcomen	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0
1773	Watt	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1

Output matrix

		a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u
1611	brick	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	coal	0	0	0	0	0	0	1	0	0	0	0	0	1	0	1	0	1	0	0	0	0
	zinc	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	copper	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
1543	iron ore	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
1500	labour	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	sand	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	potash	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	lime	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	wood	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1500	glass	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	oven	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	charcoal	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1543	iron	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	ships	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	canon	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1611	furnace	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	brass	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
1690	Savery	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
1704	Newcomen	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
1773	Watt	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1

raw materials came an increasing need for ships, and protecting these ships led to a large demand for cannons. A better, stronger cannon was developed by casting a massive cylinder and then drilling the barrel afterwards, instead of the previous method of casting a hollow cylinder. In the meantime, although raw materials were available in England, where many of these developments took place, coal and minerals had to be extracted from deeper and deeper within the earth, and mines often flooded. Initially, horses were used to haul water out of the mines, but this was rather ineffective. The first improvement was the Savery water pump of 1690 (columns `l,m`), which used steam pressure to drive water upwards. Due to the primitive construction of these early steam pumps, which lacked the required strength and precision, these systems needed much maintenance. When a leakage caused water to enter the steam chamber, a technician by the name of Newcomen was struck by the force that was generated by the condensing steam, or more precisely, the power of atmospheric pressure over an instantly created partial vacuum. In 1704 he developed this principle into a much more effective water pump, the Newcomen water pump (columns `n,o`). When Watt was asked to repair one of Newcomen's pumps, he improved its efficiency and used the improved drilled cannon barrels as a cylinder to create a more reliable and efficient steam pump (column `p,q,r,s,t`). In some cases, the water raised by these steam pumps was used to drive waterwheels. Shortly after, the waterwheel was replaced by a rod driving a crank directly (borrowed from among other things, a spinning wheel), or by a rod in combination with sun and planet gears to circumvent the crank patent, and became a power source for many processes, such as precision drilled canons (columns `u`). Eventually, coal would completely replace wood as an energy source. The importance of the steam engine as a power generator, and the impact the steam engine had on industrial development, can hardly be exaggerated. The replacement of much of the wood and water mill based industry of old by steam engines in cities, and the many spin-offs resulting from this development, is a true example of Schumpeter's gale of creative destruction. The von Neumann technology matrices are an appropriate tool to represent developments and innovations in a compact form.

Schumpeter was one of the first to realize the importance of innovation in economic development. Lacking the tools to study innovation through modelling, Schumpeter used the descriptive approach to bring the importance of capital, entrepreneurship and innovation into the spotlight. Now that we have high capacity computers at our disposal, we can attempt to mirror the innovation process and obtain an understanding of what factors affect the process and what resulting changes occur in the production system. This allows us not necessarily to predict, but to understand. In evolutionary economics there is much talk about gales of creative destruction. However, so far formal economics has not been able to simulate the process of renewal that sweeps through our economies. This thesis attempts to do just that, and expanding technology matrices are a perfect vehicle to achieve this.

4.2 Expanding technology matrices

The introduction of new technology and new products consists of two parts. First the new concepts have to be invented, subsequently, the invention has to be introduced into the production process and accepted by the market forces. This section explains the implementation of the invention process.

4.2.1 Von Neumann technology

The technology matrices of Chapter 3 had exactly as many production processes as there were products, which resulted in square matrices. A dynamic Leontief system, such as in Section 3.1.3, can easily be expanded to become a von Neumann system. In Table 4.2 for example, the technology set has been obtained by adding a column to the former Leontief technology set of Table 3.5. In this case, there are multiple techniques to produce one and the same product, and thus the number of techniques is larger than the number of products. One can choose to produce product 104 by using products 2 and 260 (columns 6), or alternatively, by using products 2, 11 and 130, which illustrates the possibility of input substitution. Furthermore, the output of the last column contains multiple products, which shows that joint production is possible. The presence of product 11 in the output is particularly interesting, since it is also part of the input required for that technique. This illustrates the implementation of capital in the system. Capital, equivalent to a catalyst in chemistry, facilitates certain processes or reactions, which would not very likely occur in its absence. Although it is possible that capital shows wear after being used, it is not completely used up. This distinguishes capital from intermediate products.

As in von Neumann (1946), it is assumed here that before prices are adapted by the price mechanism, which regulates prices during a simulation, all techniques have an equal return rate. The addition of the last columns to the input and output matrices in Table 4.2 does not require any new products. The prices of all products involved are already determined. Therefore, it is straightforward to determine the output coefficient oc such that the new column has the same return

Table 4.2: Von Neumann technology matrices with input substitution and the use of capital. *oc* stands for output coefficient and still needs to be determined. The columns in between the bars represent a new addition to the existing matrices.

products	*input matrix*								*output matrix*							
2	1	1	1	1	1	1	0	1	0	0	0	0	0	0	6	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1
13	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	1	0	0	2	0	0	0	4	0	0	0	0
260	0	0	0	0	0	1	0	0	0	0	0	0	2	0	0	0
104	0	0	0	0	0	0	1	0	0	0	0	0	0	$\frac{7}{3}$	0	*oc*

rate $1+r$ as the other columns, which is 2 in this case (Section 3.1.3). Thus from $(1+r)(1p_1 + 2p_7) = ocp_9$ and $p_1 = 1, p_7 = \frac{3}{2}, p_9 = 3$, and $1+r = 2$ it follows that the output coefficient oc is equal to $\frac{8}{3}$. Note that the return on capital (product 11) is ignored here.

4.2.2 Activity levels and price

With the appearance of an unequal number of technology columns m and product rows n ($m > n$) like in von Neumann's case, eigenvalue techniques or the Leontief inverse will no longer work to establish the price sets and activity levels. Even the existence of an unique solution is no longer guaranteed. This is where von Neumann (1946) begins.

Assume we have m production processes and n commodities in an economy, where m and n are allowed to be different. The activities are denoted by input vectors $a_i \in \mathbb{N}^n$ and output vectors $b_i \in \mathbb{N}^n$ for activity i where $i \in M$ and M is some index set of size m. We assume constant returns to scale, which means twice the input results in twice the output. The activity levels (von Neumann calls these intensities) for process i at time t are given by $z_i(t)$, and prices of the commodities are denoted by p_j, $j \in N$, where N is an index set of size n.

Von Neumann writes that he is "interested in those states where the whole economy expands without change of structure, i.e. where the ratios of the intensities $x_1 : \ldots : x_m$ remain unchanged, although $x_1, \ldots, x_m$ may change." (von Neumann, 1946, p.2). As far as this thesis is concerned, this is a severe restriction on the possible behaviour of evolving economies, but as the previous example shows, von Neumann's model can be easily adapted to include innovation. We have the following equation for the activity levels:

$$z_i(t+1) = \alpha z_i(t) \qquad \forall\, i \in M = \{1, 2, \ldots, m\}$$

Sraffa, Leontief and von Neumann regard the economy, like Quesnay and Ricardo, as a closed circular system, in which commodities as intermediates flow from producers to producers, and as consumables from producer to consumers, who are in turn producers (of labour), and so on. The sellers are also buyers, and the buyers are also sellers (Schumpeter, 1961, p.7). Prices are such that selling allows them to buy, which is necessary for continuity. This allows the economic theory to focus on only the production and flow of commodities through the economy, and it is not necessary to worry about concepts like marginal utility and its relation to price, demand and supply, which is difficult to grasp. To further avoid complication, von Neumann excludes consumption, so all produce of time t is (and must be) available for production at time $t+1$.

$$\sum_{i=1}^{m} z_i b_{ij} \geq \alpha \sum_{i=1}^{m} z_i a_{ij} \qquad \forall\, j \in N = \{1, 2, \ldots, n\}$$

In the case of excess production (strict inequality), the price of those commodities is equal to 0.

Von Neumann then assumes that no activities can have more than the standard rate of return r. If we write $\beta = 1 + r$

$$\sum_{j=1}^{n} b_{ij} p_j \leq \beta \sum_{j=1}^{n} a_{ij} p_j \qquad \forall\, i \in M = \{1, 2, \ldots, m\}$$

This means that the value of the output is less than or equal to the value of the input times $\beta = 1 + r$. In case the latter condition has a strict inequality for some i, then the activity level z_i will be 0 for that activity. Furthermore, von Neumann excludes the null solution, and thus $\forall i\ z_i \geq 0$ and at least one $z_i > 0$. The same applies to the prices: all prices $p_i \geq 0$ and at least one $p_i > 0$.

Finally, von Neumann assumes that

$$a_{ij} + b_{ij} > 0 \qquad \forall\, i \in M = \{1, 2, \ldots, m\} \text{ and } j \in N = \{1, 2, \ldots, n\}$$

and shows that under these assumption a solution exists, and in addition $\alpha = \beta$. This is quite a list of conditions, and they are not always very realistic. The last condition in particular caused much resistance among economists (Champernowne, 1946). It states that every product must be involved in every activity, either as input or as output. In the years that followed, many worked on replacing some of the conditions with more acceptable ones (Kemeny *et al.*, 1956; Gale, 1968). Kemeny *et al.* for example have replaced the last condition by the assumptions that each technique requires at least one input, and every product can be produced by at least one technique, while at least one product produced by the economic system has a nonzero price.

Based on his conditions, von Neumann stated that return rate $1 + r$ and growth factor α exist and are equal. In particular, the return rate is equal for each of the columns in the matrices. This was used in Section 4.2.1 to calculate the output coefficient of the new technology. This can be generalised as follows: suppose input matrix A and output matrix B are irreducible. Part of the input consists of capital, which is only applied and not used up during the production. Let C denote the matrix of capital output. Depending on whether we include normal wear in the model, all or part of the capital is present in the output vector. Furthermore, let p be the price vector and z be the intensity vector. Then $(A - C) \cdot p$ is the input cost for each of the production processes, $(B - A) \cdot p$ equals the profit and $\frac{(B-A) \cdot p}{(A-C) \cdot p}$ is the profit rate for each of the production processes. In order to introduce new technology or products, it is assumed that each technology has the same rate of return $1 + r$. This will allow us to determine the input and output coefficients of new columns and rows that will be added to the existing matrices.

4.2.3 New production techniques

In addition to new technology based on existing products, it is also possible to introduce new columns that make use of new products. In such a system, the new product can have three functions: it can be an intermediate product to be used as a building block for more advanced products, it can be a consumable, which is thus destined for consumption and transformed into labour, or thirdly, it can be a product that serves as capital.

As in Schumpeter (1961, p.11) and Mosekilde and Rasmussen (1986, p.30), new technology is generated and introduced at random times. With a certain probability this new technology involves the creation of new consumables. If not a consumable, the new technology aims at producing products involved in the existing production process, be it capital or intermediates. In order to make something from which the system can benefit, we introduce new technology (i.e. a tool and the skill to use it) to produce something of which there is currently a shortage, in the same way an alternative in the form of coal was found for the wood that was in short supply. Surplus and shortage are defined as in the price mechanism.

The innovation algorithm pseudo code is given in Figure 4.1. The parameters q_1, q_2 and q_3 control the probabilities of innovation and the type of innovation.

```
if random ≤ q1 innovation:
  if random ≤ q2
    then generate new consumable:
           combine random products
           set the quantity of labour resulting from consumption of this good
    else  generate new technology:
           select product that is in excess demand
           create a tool t
           if random ≤ q3
             then new tool requires new product:
                    find list of products with appropriate factorization
                    multiply these products to create new product
             else  new tool t uses old products:
                    find list of products with appropriate factorization
```

Figure 4.1: The innovation algorithm in pseudo code. *random* is a uniform random variable in the $(0, 1)$ interval. q_1, q_2 and q_3 are parameters to control the probabilities of innovation and the type of innovation.

Suppose that $random \leq q_1$ such that new technology will be generated. The case of new consumables is explained in the next section. Once one of the products in short supply is identified, the input for the new tool t can be composed. More precisely, when the product in excess demand ped is factorized $ped = f_1^{q_1} \cdot f_2^{q_2} \cdot \ldots \cdot f_n^{q_n}$ for some n and $q_. \in \mathbb{N}$, it is clear which factors are required in the input. A random list of products $i_1, \ldots, i_m$ that contain those factors is generated such that the product of these inputs $i = i_1 \cdot i_2 \cdot \ldots \cdot i_m$ will be divisible by the sought after product pcd, thus $\frac{i}{pcd} = g \in \mathbb{N}$. There are then two possibilities: a draw of a uniform random variable is less than q_3 and the tool t sets the production of the required product pcd, based on the new input i, or the uniform random variable is larger than q_3 and the tool requires the different parts $i_1 \cdot i_2 \cdot \ldots \cdot i_m$ and uses these without product i first being assembled. g is considered garbage and ignored.

Suppose we have a product set 2, 3, 5, 7, 11, 13, 70, 110, 154, 130, 260, 104. In this set 2 is labour, 3 is money, 5, 7, 11 are raw materials, as is 13, but the latter is free. The other products are composites. Furthermore, there exist production techniques to change free product 13 into consumable 104. During a simulation, a shortage of product 104 arises occasionally, and thus new technology is developed in an attempt to prevent this from happening. Table 4.3 illustrates the expansion of this existing von Neumann technology matrix. The skill columns in between vertical lines are added to the initial set of transformations.

In the example of Table 4.3, two new products are introduced, $308 = 2 \cdot 154$ and $6166160 = 2 \cdot 154^2 \cdot 130$. Tool 308 is used to transform 6166160 into product 104 of which there was a shortage. Since every process requires labour, the transformation rule of this action is $2 + 6166160 + 308 \longrightarrow 104 + 308$ as is shown in the last column. In this transformation, tool 308 is also part of the output, since the tool is merely used in the process and not used up. As can be seen, the new transformations are simply added to the existing set. With this new set of products and transformations comes an expanded price vector.

$$p = \{1, 1, \frac{21}{20}, \frac{21}{20}, \frac{21}{20}, 0, \frac{651}{200}, \frac{651}{200}, \frac{651}{200}, \frac{861}{400}, \frac{26481}{16000}, \frac{297367}{320000}, p_{13}, p_{14}\}$$

such that $((A - C) - \frac{21}{20}(B - C)) \cdot p = 0$, where, as before, A is the input matrix.

Table 4.3: The application of a new tool, where oc_1 still needs to be determined. Tool 308 is produced by the transformation in the first of the new skill columns and applied in the last column to transform 6166160s into 104s. $oc_2 = \frac{6241707}{6400000}$, such that the profit rate for all processes equals 5%.

input matrix

products	*skills*												
2	1	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	1	0	1	0	0	0	0	0	0
7	0	0	0	1	0	1	0	0	0	0	0	0	0
11	0	0	0	0	1	1	0	0	0	0	0	0	0
13	0	0	0	0	0	0	1	0	0	0	0	0	0
70	0	0	0	0	0	0	1	0	0	0	0	0	0
110	0	0	0	0	0	0	0	1	0	0	0	0	0
154	0	0	0	0	0	0	0	0	1	0	1	2	0
130	0	0	0	0	0	0	0	1	0	0	0	1	0
260	0	0	0	0	0	0	0	0	1	0	0	0	0
104	0	0	0	0	0	0	0	0	0	1	0	0	0
308	0	0	0	0	0	0	0	0	0	0	0	0	1
6166160	0	0	0	0	0	0	0	0	0	0	0	0	1

output matrix

products	*skills*												
2	0	0	0	0	0	0	0	0	0	oc_2	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	0	0	0	0	0	0	0	0	0	0	0
7	0	1	0	0	0	0	0	0	0	0	0	0	0
11	0	0	1	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	1	0	0	1	0	0	0	0	0	0
110	0	0	0	0	1	0	0	1	0	0	0	0	0
154	0	0	0	0	0	1	0	0	1	0	0	0	0
130	0	0	0	0	0	0	1	0	0	0	0	0	0
260	0	0	0	0	0	0	0	2	0	0	0	0	0
104	0	0	0	0	0	0	0	0	3	0	0	0	oc_1
308	0	0	0	0	0	0	0	0	0	0	1	0	1
6166160	0	0	0	0	0	0	0	0	0	0	0	1	0

B is the output matrix, and C is the capital matrix which has zeros everywhere except when a product plays the role of capital in a production process. This is easy to solve for the variables p_{13}, p_{14} and for the output of the new technique oc_1. The prices of the new products are determined by the cost of their input and the rate of return, which is the same for all skills. As well, the price of product 104 is already established. Since the price of the output of the new technique depends on the price of the input and on the output quantity, and the prices are set, there is a unique solution for oc_1. As in von Neumann's theory, the rate of return on input for each of the processes is equal, and here is 5%.

In the case that the random variable *random* in the innovation algorithm of Figure 4.1 is larger than q_3, then the input for the new technology is a list of old products instead of the new product 6166160. This would lead to two new columns instead of three new columns as was the case in Table 4.3. The first new column below shows the production process to make the tool, and the second new column shows the use of the tool applied to a number of products in order to generate product 104. The output coefficient oc can be determined as above.

$$\{1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0\} \rightarrow \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1\}$$

$$\{1, 0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 1\} \rightarrow \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, oc, 1\}$$

On a side note, it is also possible to reduce the price of product 104. By doing so, the new production process would be able to earn the standard interest rate of 5%, while the old production process is reduced in its return rate because of the lower price. The older technique could still make a profit, but in equilibrium, it will be less than the standard return rate. Its services are possibly still required by the system.

even though the old process now shows a lower performance. The success of the new technique will depend on its introduction by innovating agents (see Section 4.3). It is possible that the new technology is a striking success, but until the new technology takes over completely, the old system that is already in place can keep up with the demand that is not met by the new technology. This would lead to a gradual introduction of the new technology, comparable to the dynamics resulting from a control parameter in Morgenstern and Thompson (1976, section 8.3). The difference is that in the agent based model, it is not the modeller who slowly increases the parameter to raise the market share of the new technology. It is instead the agents themselves who decide whether or not to innovate, and who control what part of the market is covered by which technique. It is possible that more than one technique is used, and these may not even be the most efficient ones. This is a very realistic situation: Dosi *et al.* write that the persistence of asymmetric performances is one of the puzzles in economic studies yet to be explained (Dosi *et al.*, 1995). The idea of different return rates has not yet been explored, but even without different return rates there will appear differences in the success of the technologies. The model results will show interesting competition between techniques producing the same product.

4.2.4 New consumables

Adding a consumption good is less complicated than adding capital or an intermediate product. A new product is created and introduced as described above, and this new product can be consumed in order to produce labour.

The first column of Table 4.4 shows the production of a new product $1820 = 2 \cdot 7 \cdot 130$, and the second column shows the conversion of this product into labour.

Table 4.4: Additions to the existing technology matrices of Table 4.3 to introduce a new consumable. The first of the new columns of the input matrix supplement shows the input for product 1820. The second of the new columns of the input and output matrices shows the consumption of oc_3 units of 1820 to result in oc_1 units of labour (product 2).

	new columns input matrix		**new columns output matrix**	
products	*skills*		*skills*	
2	1	0	0	oc_1
3	0	0	0	0
5	0	0	0	0
7	1	0	0	0
11	0	0	0	0
13	0	0	0	0
70	0	0	0	0
110	0	0	0	0
154	0	0	0	0
130	1	0	0	0
260	0	0	0	0
104	0	0	0	0
308	0	0	0	0
6166160	0	0	0	0
1820	0	oc_3	1	0

Again, the values of p_{15}, oc_3 and oc_1 have to be determined such that the profit rate is 5%. Consumers can be modelled such that they do not have a preference for any of the consumables available: they will use whatever is available at the smallest cost. Another option is to let consumer preference correspond to the labour output coefficient oc_1. More advanced products can thus be more attractive to the consumer than old fashioned products. This is not the most elegant model of consumer demand, but consumer preference is a very difficult subject and falls outside the scope of this research. For an introduction to consumer behaviour modelling see Aversi *et al.* (1999), which develops a probabilistic consumer model based on imitation or innovation of consumers' preference strings. The next chapter will discuss some outcomes of different modes of consumer behaviour.

In principle, the model can expand the matrices indefinitely. However, this is, of course, impossible due to practical constraints. Instead of immediately replacing existing production processes with new ones, as is done in Jain and Krishna (2002), the model adds columns to the matrices, but also keeps track of the use of the different skills. As well, the model keeps a record of the performance of the agents. Poorly performing agents and obsolete skills are removed, and sometimes this allows for products to be removed, since they have become obsolete.

If successful, the introduction of new techniques, which will be dealt with in the next section, possibly reduces the need for older processes in the technology set even though the new technology has the same equilibrium profit rate as all the other processes. When competing technologies (i.e. technologies making the same product) meet with different levels of success, often the difference can be explained by the more efficient production of advanced techniques. This is caused by more intensive production, a higher output per unit of action. It is therefore to be expected that over time, different sets of active production techniques will appear. The introduction of new technology can therefore make older technology obsolete. Older technologies can be removed from the system by deleting columns in the input and output matrices. This in turn can lead to a reduced demand for some inputs, and their production skills. Such a continuous process of replacement of products by new products and technology by new technology brings us to the Schumpterian gales of creative destruction.

4.3 Entry and exit of technology

As discussed in Section 3.2.3, the agent's behaviour is determined by maximization of the profit generated by the agent's actions, to which certain constraints are applied. These constraints deal with available means and commodities, as well as with the skills the agent possesses. With the expansion of the technology matrices as in the previous section, new products and new skills are not yet introduced into the economy. The new products only appear when agents use the new skills and the new products. This section describes the process of the introduction and the diffusion of skills. The model design of this process is based on Bruckner *et al.* (1989).

4.3.1 Technology fields

Bruckner *et al.* (1989) describe a general model to study evolutionary processes. The model consists of a countable set of fields $F = \{F_1, F_2, \ldots\}$ with, for each field F_i, a number N_i indicating the number of elements in the field. The state of the whole system is given by the set of occupancy numbers $\{N_1, N_2, \ldots\}$. Changes in occupancy numbers are discrete and occur in the smallest steps possible: fields gain or lose one element. The probabilities of these changes depend on the current state of the system.

The transition probability W of field i increasing in size depends on self-replication, second order self-reproduction, and growth sponsored by other fields.

$$W(N_i + 1|N_i, N_j) = A_i^{(0)} N_i + A_i^{(1)} N_i N_i + B_{ij} N_i N_j$$

where $A_i^{(0)}$ is the parameter controlling self-reproduction, $A_i^{(1)}$ regulates the probability of self-amplification, and B_{ij} deals with sponsoring of one field F_i by another field F_j. Besides these events driven by the current state, the size of field F_i may increase due to some spontaneous event, such as in-migration, and the fixed probability is determined by parameter ϕ_i.

$$W(N_i + 1) = \phi_i$$

The size increment may occur because of mutation. The parameter M_{ij} controls the probability that element i results as a mutation from element j. The probability of error reproduction is given by the expression below, where again the larger the field F_j, the larger N_j is, and thus the larger the probability of a mutation originating from F_j.

$$W(N_i + 1, N_j | N_i, N_j) = M_{ij} N_j$$

The expression for a decrease in size is comparable to that for an increase in size. $D_i^{(0)}$ represents the linear death rate, $D_i^{(1)}$ and D_i^2 are the non-linear self-inhibition parameters, and C_{ij} controls how field F_j constrains field F_i. The probability of the death of an element is given by

$$W(N_i - 1 | N_i) = D_i^{(0)} N_i + D_i^{(1)} N_i N_i + D_i^{(2)} N_i N_i N_i + C_{ij} N_i N_j$$

It is also possible for fields to exchange an element. The fields involved in this process can cooperate, or the exchange can occur because a field expels an element into another field. The probability that one element of field F_j moves to field F_i is

$$W(N_i + 1, N_j - 1 | N_i, N_j) = A_{ij}^{(0)} N_j + A_{ij}^{(1)} N_i N_j$$

where $A_{ij}^{(0)}$ controls the probability of uncooperative growth of F_i, i.e N_i is not involved in this process, and $A_{ij}^{(1)}$ is the parameter for cooperative exchange.

The Markov process given by this set of probabilities of interactions between fields is capable of generating a wide variety of evolutionary dynamics (Bruckner *et al.*, 1989). In particular, Bruckner *et al.* write that this type of model is capable of simulating the dynamics of evolutionary systems, including the dynamics of technological evolution, for which they refer to Mosekilde and Rasmussen (1986), who have applied a similar approach to study economic succession and waves. In a later paper, Bruckner *et al.* experiment with the parameters for the stochastic economic substitution model and show that realistic substitution dynamics can be obtained (Bruckner *et al.*, 1996).

In the context of this thesis, a field represents a technology, and the occupancy of a technology is the number of agents that use that technology. The stochastic process describes the interaction between the fields, where, in the general model, the possible interactions are self-reproduction, exchange, decline, and spontaneous generation. In evolutionary economics, one can think of the creation of new agents, with new or existing technology, innovation by an existing agent by the acquisition of new technology, imitation of successful technology, bankruptcy, and spontaneous introduction of new technology by in-migration of new economic agents.

The model can be used here to regulate the substitution of old techniques and products with new ones. Our model does not merely give the size dynamics of the different fields. Instead, the interaction between fields is used to regulate the introduction and removal of agents and technology. Consequently, we are not so much interested in the size of the fields as we are in what changing occurrences of technology mean for the market dynamics, which obviously are affected by changing technology, products and economic agents. In order to apply the Bruckner *et al.* framework for field interaction to the substitution of technologies, we need to determine a reasonable set of parameters $A_i^{(0)}, A_i^{(1)}, B_{ij}, M_{ij}, D_i^{(0)}, D_i^{(1)}, D_i^{(2)}, C_{ij}, A_{ij}^{(0)}, A_{ij}^{(1)}, \phi_i$ to control the probabilities of events, such as the creation of new agents, innovation, imitation, and the removal of agents.

4.3.2 Probabilities

As mentioned above, Bruckner *et al.* (1996) describes a similar exercise. Bruckner *et al.* distinguish a number of different innovative processes, and not all of these apply to the model in this thesis. In particular, they distinguish firms and plants, where one firm can consist of different plants, and plants are production units of firms. In this thesis, every production unit, i.e. every agent, is independent and considered a firm. When Bruckner's list of innovation processes is limited to the ones that are applicable to the model here, we obtain a list of five relevant types of events: innovation by a new agent, innovation by an existing agent, random imitation of technology, spawning of a new agent with existing technology, and imitation of successful technology by an existing agent.

These types of events can be interpreted in such a way that they fit within the general framework provided by Bruckner *et al.* (1989).

Innovation by new agent: When a new technology becomes available, its establishment is affected by some of the existing technologies. Parameter A_{ij} describes the inclination of technology j to establish technology i by means of a new agent: $W(N_i + 1, N_j | N_i = 0, N_j) = A_{ij} N_j$

Innovation by existing agent: When an agent expands its skills, be it with a new technology or an existing technology, the agent innovates its production process. The choice of additional technology is affected by the existing technologies. Parameter M_{ij} describes the inclination of technology j to establish technology i by means of an existing agent: $W(N_i + 1, N_j | N_j) = M_{ij} N_j$

Growth of existing technology by spontaneous new agent: An increase in the number of agents using technology i independent of the state of the system: $W(N_i + 1) = \phi_i$

Growth of existing technology by new agent: An increase in the number of agents using technology i due to self-reproduction or sponsoring by technology j: $W(N_i + 1, N_j | N_i, N_j) = A_i^{(0)} N_i + A_i^{(1)} N_i N_i + B_{ij} N_i N_j$

Replacement of technology by existing agent: Agents imitate the successful technologies of other agents, and thus replace less successful skills. The parameters A_{ij} represents a measure of success and failure. Furthermore, the probability is influenced by the current size of the the technology fields. The larger the field, the more occurrences of the technology in question, and the greater the probability of replacement: $W(N_i + 1, N_j - 1 | N_i, N_j) = A_{ij}^{(0)} N_j + A_{ij}^{(1)} N_i N_j$

An important type of event is missing from this list. In the present model the removal of technology from an agent is not probabilistic, as is the case in Bruckner *et al.* (1989, 1996). Here we have chosen to remove technology when it is not used during a simulation. Removal of unused technology, and subsequently, unused products, is realistic and in addition has the advantage of keeping the program running at a reasonable speed as much as possible.

The quantification of the parameters to determine the above transition probabilities is a free interpretation of Bruckner *et al.* (1996), who base the definition of parameters on work by Nelson and Winter (1982). All agree that since technology exists by the grace of agents or firms applying the technology, and since these firms operate in a market, parameters governing the dynamics of technology fields (i.e. changes in the number of occurrences of technology) need to be based on economic indicators. Three indicators are used to define the parameters: distance between technology, gross return, and connectivity, and these economic indicators influence the probabilities of changes in the occurrence of technologies in the model.

Nelson and Winter define a measure for distance between technologies: the closer two technologies are, the more comparable they are, and the easier it is for an agent to change from one technology to the other (Nelson and Winter, 1982, p.211). In their case, technology is given by two output coefficients, one for capital and one for labour. We can do something similar by defining the distance between two technologies i and j based on input coefficients $c_{i1}, \ldots, c_{in}$ and $c_{j1}, \ldots, c_{jn}$ respectively by:

$$d(i, j) = \exp(-\sum_{k=1}^{n} |c_{ik} - c_{jk}|).$$

This results in an inverse distance measure for technologies, where for technologies with equal input coefficients the inverse distance is equal to 1. For example, the distance between the first two columns in Table 4.2, $\{1, 0, 0, 0, 0, 0, 0, 0, 0\}$ and $\{1, 0, 0, 0, 0, 0, 0, 0, 0\}$ respectively, is $\exp(-|1-1|) = \exp(0) = 1$. The distance between the fifth column and the last column in that table, $\{1, 0, 0, 0, 0, 0, 1, 0, 0\}$ and $\{1, 0, 0, 0, 1, 0, 2, 0, 0\}$ respectively, would be $\exp -(|1-1| + |0-1| + |1-2|) = \exp(-2) \approx 0.135$. With increasing differences in coefficients, the inverse distance will decrease to 0. This measure can be used to define the probability of substituting one technology for another.

Bruckner *et al.* suggest that gross return is an appropriate measure to distinguish between the capability of technologies to expand their field. To measure the actual gross return per technology, activity levels per technology of the last w iterations are added and multiplied by their output value to calculate total revenue per technology. Note that this measure is dependent on time t, since production levels will vary over time.

$$gr(t) = (total\ production(t) - total\ production(t - w)) \cdot \frac{\{o(1), \ldots, o(n)\}}{\{N_1, \ldots, N_n\}}$$

Here $o(s)$ is the value of the output of skill s, and thus, gross revenue $gr(t) = \{gr(1), \ldots gr(n)\}$ is a vector of values of output generated by each of the technology per agent possessing that technology. Subsequently, the vector is normalized by division by the largest element, resulting in a vector of elements larger than or equal to 0, and smaller than or equal to 1, with 0 indicating no output, and 1 indicating the largest output per occurrence of that technology.

Finally, we define a measure to indicate the connectivity of different technologies. To simulate vertical expansion of an agent, it is interesting to take into account the connections sectors have, either to upstream supplying technologies or to downstream buyers that are being served by the sector (or both). This measure is given by the boolean input-output matrix to indicate the existence of flows between the different sectors. It is straightforward to construct a square matrix of size n, where n is the number of technologies in the model. For example Table 4.5 gives the upstream connections of the technologies. The transpose of this matrix gives the downstream connections. The sum of these two matrices represents all connections.

The coefficients used to determine the transition probabilities are based on these three economic indicators. The sponsoring of new technology by existing technology

Table 4.5: Upstream connections between the technologies in Table 4.2

		buying sector							
		1	2	3	4	5	6	7	8
selling sector	1	0	0	0	1	0	0	0	0
	2	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	1
	4	0	0	0	0	1	0	0	1
	5	0	0	0	0	0	1	0	0
	6	0	0	0	0	0	0	1	0
	7	1	1	1	1	1	1	0	1
	8	0	0	0	0	0	0	1	0

depends on the connections between the new and old technology. An existing technology will sponsor the establishment of a new technology only if the new technology deals with either input or output of the existing technology, regardless of whether the new technology is established by an existing agent or through a new agent. The sponsoring by existing technology of a new agent with a particular technology (new or existing) depends on all three factors: the gross return of the sponsoring technology, the distance between the two technologies in question, and the connections between the two. Finally, the imitation probabilities depend on how poorly one technology is doing and how well another technology is performing. The lower the gross return, the higher the coefficient to replace the poorly performing technology with another technology. The higher the gross return of a technology, the higher the coefficient of substituting that technology for something else. Table 4.6 provides an overview of the different transitions and their dependencies. Note that some parameters are set equal to 0.

Once the dependencies of innovating events are quantified, these values can be easily transformed into a probability distribution that describes the introduction and

Table 4.6: The definition of entry and exit coefficients. d stands for the distance matrix, gr stands for the gross return of each of the technologies, and c stands for the connection matrix. f and g are functions combining the economic indicators into one matrix. For example, g combines the gross return $(1 - gr)$ and gr to define combinations of poorly functioning technologies with well functioning technologies in order to find appropriate replacements for poorly performing technologies. N_i is the number of occurrences of technology i in the model, i.e. the size of field i.

Type of innovation	Dependency
Innovation by new agent $W_1(N_i + 1, N_j \| N_i = 0, N_j) = M_{ij} N_j$	$M_{ij} = c$
Innovation by existing agent $W_2(N_i + 1, N_j \| N_i = 0, N_j) = A_{ij} N_j$	$A_{ij} = c$
Spontaneous new agent $W_3(N_i + 1) = \phi_i$	$\phi_i = Random(0, 1)$
Expansion through new agent $W_4(N_i + 1, N_j \| N_i, N_j) =$ $A_i^{(0)} N_i + A_i^{(1)} N_i N_i + B_{ij} N_i N_j$	$B_{ij} = f(gr, d, c),\ A_i^{(0)} = 0,\ A_i^{(1)} = 0$
Imitation $W_5(N_i + 1, N_j - 1 \| N_i, N_j) =$ $A_{ij}^{(0)} N_j + A_{ij}^{(1)} N_i N_j$	$A_{ij}^{(1)} = g(gr),\ A_{ij}^{(0)} - 0$

removal of agents and technology. Suppose there are n different technologies. For each of these n technologies there are five different types of transitions. For only one of these types are the coefficients independent of the state of the system, namely the spontaneous introduction of an agent with technology. The transition coefficients for spontaneous introduction are given by n randomly generated coefficients: one for each of the technologies, such that different technologies can have different probabilities to be introduced at random. The other types of transition depend on both the receiving technology i, as well as the technology j the transition originates from (sponsoring or replacement), and thus, the transition coefficients are given per pair of technologies, n^2 for each type of transition. The example in Figure 4.2 is generated with a total of $n = 12$ technologies, of which one is new, which implies that innovation can occur since one field has size 0. Based on twelve technologies, the sponsored creation of a new agent with a particular technology can occur in n^2 ways, 144 in this case. The same applies to innovation through a new agent, innovation by an existing agent and the replacement of a technology by another technology. The total number of events is thus 144+144+12+144+144=588, and for each of these events, a coefficient has been calculated according to the rules in Table 4.6. All coefficients $W_1(i,j), W_2(i,j), W_3(i), W_4(i,j)$, and $W_5(i,j)$ can be divided by the total $\sum_{i,j-1}^{n} W_1(i,j) + W_2(i,j) + W_3(i) + W_4(i,j) + W_5(i,j)$ in order to create the probability distribution. However, in order to guarantee equal total probabilities for each of the types of transition, the coefficients are first summed and divided by the total per type. This process leads to the cumulative transition probability distribution in Figure 4.2. The figure shows 588 intervals, although many of the intervals have size 0 (where there is no increment). A random number in the range (0, 1) determines which event will occur. A new agent with a technology indicated by the random draw will occur on a random location on the map. Imitation occurs when a technology field F_i gains at the expense of another field F_j ($N_i + 1, N_j - 1$). For this to occur in the agent based model, an agent that possesses the technology j and not i is selected; that agent looses technology j and gains technology i. To implement the introduction of innovation i sponsored by technology j by an existing agent, an agent is chosen such that that agent possesses the sponsoring technology j and this agent acquires technology i. This leads to a random process implementing the technology field dynamics, see Appendix B.6 for the scheduling of these events.

Note that this model functions on both a global and a local level. Global in-

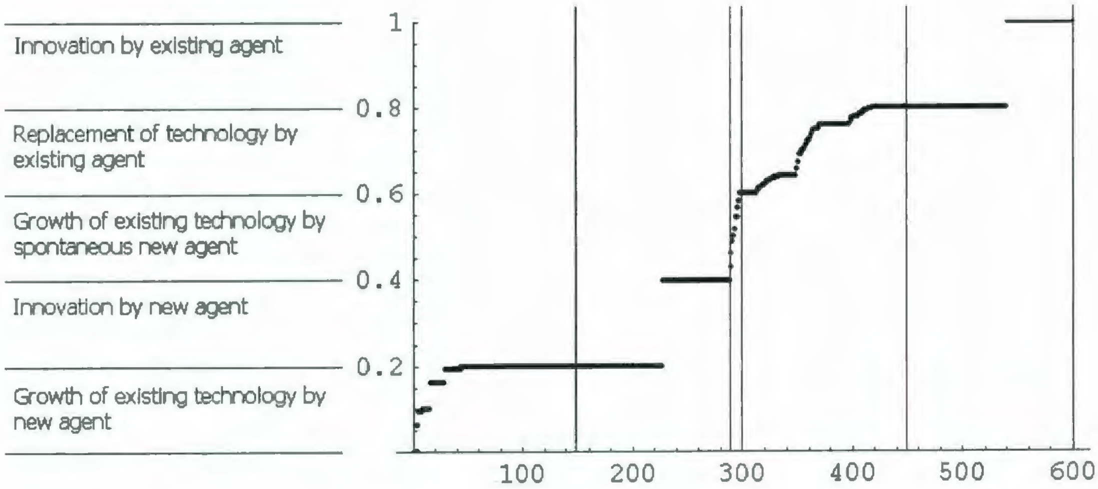


Figure 4.2: The cumulative distribution of transition probabilities. The model allows for five types of transition, and each of these types has an equal probability of occurrence.

formation, such as gross return of a sector, is used to generate the probabilities of introducing new agents into a sector. Local information, such as bankruptcy, determines the removal of agents. It is possible to let the agents have their own learning strategies, based only on local information. This would be more in line with Nelson and Winter (1982) and other evolutionary economics research, which focuses on bounded rationality and learning to model evolving technology. Adaptation of skills based completely on local and bounded information has not yet been explored by the model in this thesis. Here we focus on changes in the flows as a result of technological change.

Chapter 5

A simulation

A simulation of an economy consisting of spatial agents and a fixed product and technology set, as described in Section 3.3, can run indefinitely, but the question this thesis sets out to answer is what happens when new technology or new products are generated by the existing system? The previous chapter has described a model of the innovation process, and allows us to study the consequences of the introduction of new technology. It is here that this research extends the existing body of economic geography and economics, by treating the spatial economy as if it were an open dimensional open thermodynamic system. Such a system tries to find ways to deal with the surplus of energy that is fed into it, and that dissipates into all sorts of products and activities, while allowing creativity on behalf of the actors, who find more and more advanced ways of guiding energy flows to the sink. As a result, more elaborate products and production processes appear, and it is this emergence of structure that this work tries to capture.

Obviously, it is impossible to predict the specifics of real economic evolution in the model. In biology, where evolution has been studied for much longer and to a much greater extent than in economics, it is unknown where evolution will take the biosphere, including ourselves. This is not what biology is aiming for though:

instead, it intends to provide understanding of the process by which evolution takes place. The situation is similar in economics, and specifically in this model. It is unknown where evolution is going to take the economy. Nevertheless, simulating an evolving economy provides insight into the possible consequences of innovation or changing boundary conditions. The present chapter discusses a typical simulation run of the model developed in this thesis. It explains the setup of a model run and the events that occur during the run. The general results of such runs will be discussed in more detail in the following chapter.

5.1 The model setup

The agents in this simulation are located on a bounded grid. At random locations, two agents are placed; one agent is a producer, and the other is a consumer. For example, the location $\{2, 4\}$ contains the following two lists of numbers:

$$\{\{2, 4, 1\}, \{0, 100, 0, 4, 4, 2, 2, 2, 2, 1, 1, 1\}, \{1, 1, 1, 0, 0, 0, 1, 0, 1, 0\}\}$$

$$\{\{2, 4, 2\}, \{0, 100, 0, 4, 4, 2, 2, 2, 2, 1, 1, 1\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, 1\}\}$$

These lists indicate that the location contains two agents, one agent with identity $\{2, 4, 1\}$ and one agent $\{2, 4, 2\}$. Initially, all agents receive an equal, small amount of resources and 100 units of money. In this case, the list of resources is given by $\{0, 100, 0, 4, 4, 2, 2, 2, 2, 1, 1, 1\}$. The final boolean list for each of the agents identifies which skills or technology the agent possesses. Here there are 10 skills, and agent $\{2, 4, 1\}$ possesses skills 1,2,3,7, and 9. The second agent at $\{2, 4\}$ only has skill 10. The skills represent the transformations indicated in Table 5.1.

These matrices show the input and output required for all possible transformations, where each column represents one transformation or one technology. Input

Table 5.1: The possible transformations, where all sectors have an equal profit rate of 100%.

products	*input matrix*										*output matrix*									
2	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	3
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
11	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
13	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	$\frac{1}{5}$	0	0	0
110	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	$\frac{1}{5}$	0	0
154	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	$\frac{1}{5}$	0
130	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	3	0	0	0
260	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	3	0	0
104	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	4	0

and output are given by units of products, and the different products are listed in the left most column. Here, the initial list of products is

$\{2, 3, 5, 7, 11, 13, 70, 110, 154, 130, 260, 104\}$,

where 2 is labour, 3 is money, and $5, \ldots, 13$ are raw materials, of which 13 cannot be produced and is freely available. The other products are composites of these primitive products. Note that labour (product 2) can be generated by transforming product 104 into 2, represented by the technology in the last column of the input and output matrices. All agents possessing this skill are consumers, while the other skills belong the the producers.

The output coefficients in Table 5.1 have been chosen such that the largest real generalized eigenvalue of the input matrix, with respect to the output matrix, equals $\frac{1}{2}$. This results in both a growth rate and a return rate of 2. A return rate of 2 corresponds to 100% return on investments. This is not a realistic value, but it allows the resulting output coefficients to be more user friendly, i.e. integers or simple fractions instead of fractions with a large numerator and denominator. That the growth rate of the system shown in Table 5.1 is in fact 2 can be verified by the fact that the activity vector $z = \{8, 0, 0, 0, 0, 0, 4, 6, 9, 18\}$ requires $input\ matrix \cdot z = \{27, 0, 4, 0, 0, 4, 4, 6, 9, 6, 9, 18\}$ and results in $output\ matrix \cdot z = \{54, 0, 8, 0, 0, 0, \frac{16}{5}, \frac{24}{5}, \frac{36}{5}, 12, 18, 36\}$. When labour is taken as the numeraire good with a price of one, the resulting price vector is $\{1, 1, 2, 2, 2, 0, 10, 10, 10, 2, 2, \frac{3}{2}\}$, and

$$\frac{p \cdot (output\ matrix - input\ matrix)}{p \cdot (input\ matrix - catalysts)} = \frac{\{1, 1, 1, 5, 5, 5, 3, 3, 3, \frac{3}{2}\}}{\{1, 1, 1, 5, 5, 5, 3, 3, 3, \frac{3}{2}\}}$$

In other words, the profit (output minus input) per technology, divided by the input costs, disregarding the catalysts, equals 1 for each of the technologies, a 100% return on the input, so to speak. As explained in Chapter 4, the constant return rate is used in a later stage used to determine the output coefficient of new technology discovered during the run of the simulation.

In this simulation, agents are placed randomly in pairs on a grid. In this particular case, this results in 29 locations with agents, as shown in Figure 5.1. The dots represent locations where agents are present, and the lines connect locations such that agents can interact with surrounding agents. Locations without a dot harbour no agents.

The network of connections is not fixed. At random an agent drops one of its furthest connections, scans the neighbourhood and adds the nearest supplier of a good that is lacking in the local market. The network one iteration later is displayed in Figure 5.2.

The rules that govern the behaviour of agents were described in Section 3.2.3. It was explained that every agent maximizes the value of its possessions through

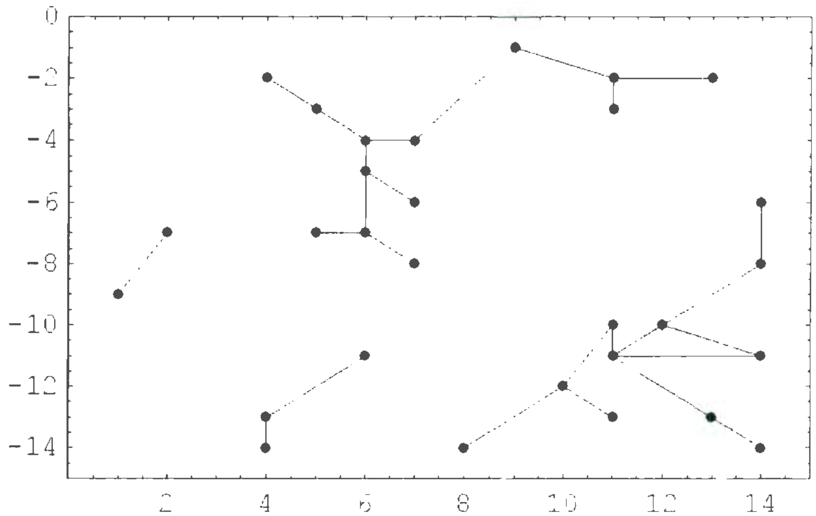


Figure 5.1: The agents on a map. The dots represent the locations where agents are present, and the lines connect locations such that agents can interact with surrounding agents.

engaging in profitable activities. However, in doing so, a number of constraints have to be taken into account. When setting up a simulation, parameters controlling these constraints have to be set. Appendix A provides an overview of the parameters involved in the simulation described in this chapter.

By taking into consideration these constraints and current commodity prices, each producer maximizes its possessions. This will alter the activity record and the goods in store, and thus the prices may have to be adapted to reflect the new supply and demand. New prices will lead to new profitability levels, and as a consequence, the agents will adapt their behaviour.

In this simulation, the model does not have a exogenously fixed consumption rate c for all consumers as was used in Section 3.3. Instead, each consumer individually decides how much it can afford. Consumers aim to maximize the quantity of labour they can produce. Initially, there is only one consumable, product 104 in Table 5.1,

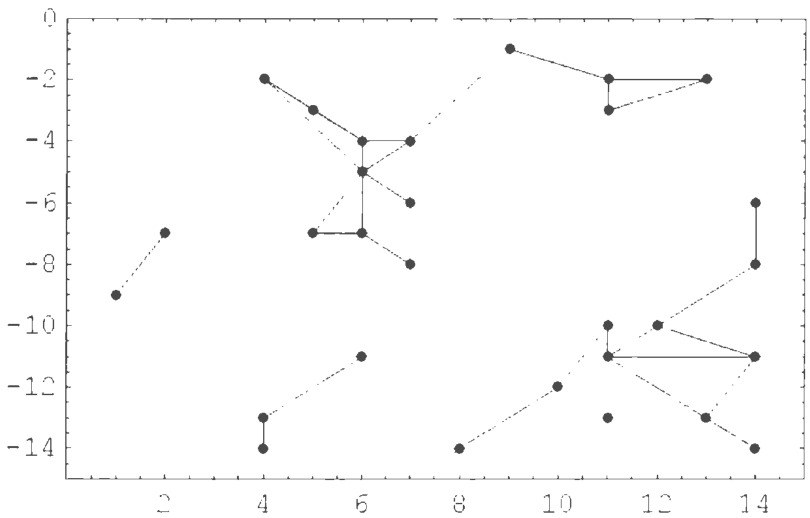


Figure 5.2: The agents in space with connections

and thus they buy as much of it as they can afford and transform it into labour, which will be available to the producers as one of the inputs in their manufacturing processes. Later in the simulation, more advanced consumables will be available. These products enable the consumer to generate more labour per unit input, but they come with a higher price as they require more advanced inputs to be produced. Currently, consumer preference is set by the output coefficient; the greater the amount of labour generated through the consumption of a particular product, the more desirable the product is. There is also the possibility of experimenting with more elaborate models of consumer preferences here, but so far this has not been done. Labour generated by the consumers is available to the producers.

The income of all agents consists of the revenue generated by their production of goods or labour. Consumers, however, have an additional source of income in the form of dividend. When producers make a profit due to the sale of goods, because the input costs to produce these were lower than the value of the output, the surplus revenue is equally divided over the consumers, as if they all had an equal stake in the production process.

To begin the simulation, all agents are randomly ordered in a list; each of the agents is selected in turn and goes through the process described in Section 3.2.3. Before an agent makes its plan for producing, buying and selling, prices are set based on the agent's local market situation. Then the agent wins free resources, possibly replaces a distant trade partner with a nearer one, makes a plan that optimizes possession, buys missing ingredients, sells surplus of products to generate income, and engages in production of the profitable products by executing the corresponding techniques, as well as engaging in the transformations that are not profitable, but that are nevertheless necessary to enable the execution of the plan. All activities are logged, and the simulation moves on to the next agent on the list. When all agents have had their turn, one iteration has been completed, and the process described in this paragraph is repeated.

5.2 Model behaviour

5.2.1 The initial fixed run

During the simulation, all transactions and production activities are logged. An example of this is given in Figure 5.3. This fragment of the log book shows the actions of five agents during the simulation. The first agent {14.4.1} finds itself in a situation in which taking no action results in the best outcome. None of the technology available to this agent is profitable, or if it is, the resources required are not available. The agent simply passes its turn, and the simulation moves on to the the next agent at location {14.14.1}. This agent is in the position to generate profit, and it executes skill 8 and skill 9, 0.5 and 3.5 times respectively. Note that these

```
initiative in step 64 taken by
{14, 4, 1}{0., 97.5, 4., 4., 4., 3., 2., 2., 2., 1., 1., 0.}
At step 64 end result of agent {14, 4, 1} is
{0., 97.5, 4., 4., 4., 3., 2., 2., 2., 1., 1., 0.}

initiative in step 65 taken by
{14, 14, 1}{0., 100., 0., 4., 4., 6., 2., 2., 2., 1., 1., 0.}
execute 0.5 times skill 8
execute 3.5 times skill 9
4. money for 4. pieces of 2
15. money for 1.5 pieces of 154
2. money for 1. pieces of 260
At step 65 end result of agent {14, 14, 1} is
{0., 79., 0., 4., 4., 6., 2., 2., 3.5, 0.5, 0., 14.}

initiative in step 66 taken by
{4, 7, 1}{0., 100., 0., 4., 4., 12., 2., 2., 2., 1., 0., 1.}
execute 1. times skill 9
1. money for 1. pieces of 2
2. money for 1. pieces of 260
At step 66 end result of agent {4, 7, 1} is
{0., 97., 0., 4., 4., 12., 2., 2., 2., 1., 0., 5.}

initiative in step 67 taken by
{14, 8, 2}{2., 103.356, 0., 4., 4., 5., 2., 2., 2., 1., 1., 0.}
At step 67 end result of agent {14, 8, 2} is
{2., 103.356, 0., 4., 4., 5., 2., 2., 2., 1., 1., 0.}

initiative in step 68 taken by
{7, 2, 2}{2., 103.356, 0., 4., 4., 8., 2., 2., 2., 1., 1., 0.}
execute 1. times skill 10
2.455 money for 1. pieces of 104
At step 68 end result of agent {7, 2, 2} is
{5., 100.901, 0., 4., 4., 8., 2., 2., 2., 1., 1., 0.}
```

Figure 5.3: Logging of transactions and production

activity levels sum up to 4, which is the maximum total activity level per agent. In order to execute these skills, it requires additional resources, which are bought from surrounding agents. The money and commodities are exchanged, and, at the cost of 4+15+2=21 units of money, 14 units of product 104 are produced. As the end result of these actions, this agent is down 21 units of money in comparison with its initial state and up 14 units of product 104. The next agent {4,7,1} shows similar behaviour.

The last two agents shown in the log in Figure 5.3 are consumers. Agent {14,8,2} begins its turn with 2 units of labour and sufficient monetary resources, but apparently has no access to consumables and thus remains in the same state. The last agent in the above illustration is able to buy one unit of product 104, and subsequently executes skill 10 to generate three new units of labour.

5.2.2 Acquiring additional skills

After a number of iterations, the initially abundant stocks of resources are depleted, and the agents have to produce the full gamut of products in order to supply the demand for consumable 104. Because the skills were randomly chosen, the initial distribution of skills is not ideal. With the technology field model described in Section 4.3, the agents will slowly replace less frequently used skills with more successful skills, or adopt new skills that will support skills they already possess. The following paragraphs describe the implementation of this process, and subsequently, the parameters controlling the technology field interactions are discussed.

Figure 5.4 shows the production of the different products for each iteration from the start of the model run. Because the price mechanism depends on the history of the system, and the past is non-existent at the start of the simulation, an empty history is given. This is visible in the graphs as no activity during the first 50 iterations. However, the consumers then begin to demand their product 104 and to convert this product into labour. There are some products 104 in the system at the outset, but these are soon consumed and thus, the stock is smaller than the demand. This will lead to an increased price for product 104. All agents with skill 9 (column 9 in Table 5.1) see the opportunity to make a profit, and thus start demanding product 260, and, in smaller amounts, product 154, which are, respectively, the input and capital involved in the production process of product 104. Shortly after, the stock of 260 is depleted or smaller than the demand, and agents with skill 8

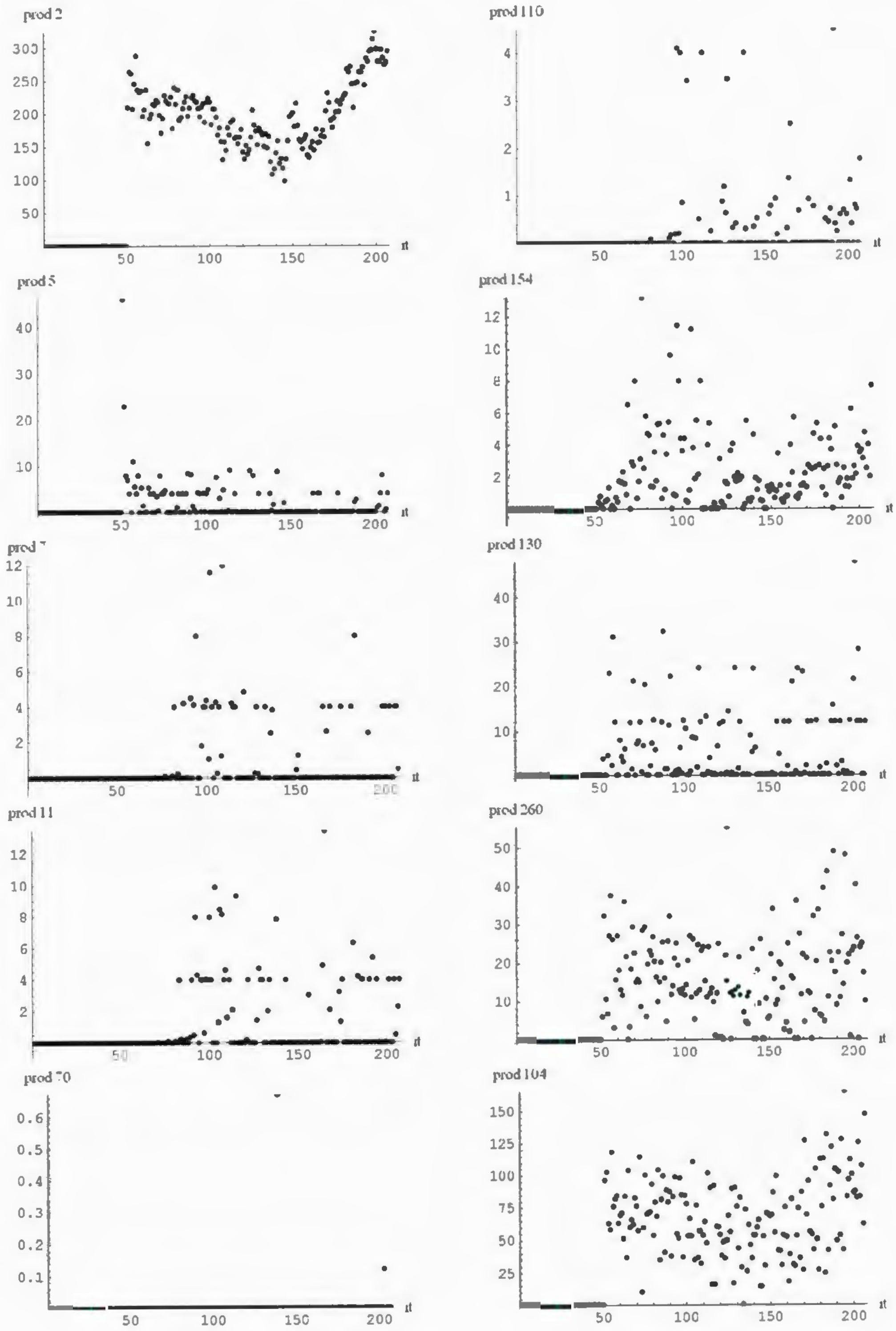


Figure 5.4: Production of capital and labour per iteration. During the first 50 iteration are there is no activity because the price mechanism depends on the history of the system, and the past is non-existent at the start of the simulation, an empty history is given.

come into action, which leads to demand for products 130 and 110, the input and the capital required, respectively, for product 260. Now, the same happens again to other products further down the production line, and thus the demand for the consumable 104 cascades down, eventually leading to demand for the raw materials.

Product 2 (labour) is the end product, and, at the same time, the beginning of all action. It is labour that enables agents with skills 1,2 or 3 to mine raw materials. Therefore, the level of labour production is a good measure for assessing the activity level in the system. As can be seen in Figure 5.4, labour production begins high, and remains more or less constant until iteration 100. At this point it falls back a little, after which it gradually regains higher levels. The higher starting level is explained by the abundant stocks of resources handed out in the setup of the system that can be used when convenient. When these resources are used up, all resources have to be produced, and the agents' activity has to be divided over all necessary processes. This explains the slight reduction of labour production around iteration 100, due to the lack of product 104 and its ingredients.

At this time, the model begins to implement the interaction between the technology fields. In other words, agents begin to learn, or to replace less needed skills with more successful skills. The fact that labour production slowly increases again can be explained by the slow change in the distribution of skills. The probabilities of changing skills are controlled by the parameters in Table 5.2

The parameters in this table differ from the ones given in the previous chapter. Here, the parameters are set such that there are very few introductions of new agents to the system. The only time this occurs is when a new technology is established by a new agent. New technology can also be established by an existing agent, and the

Table 5.2: The definition of entry and exit coefficients. c stands for the connection matrix, n for new technology, o of the occupancy rate, and gr stands for the gross return of each of the technologies. f g and h are functions combining the economic indicators into one matrix. N_i is the number of occurrences of technology i in the model, i.e. the size of field i.

Type of innovation	Parameters
Expansion through new agent $W(N_i+1, N_j \mid N_i, N_j) =$ $A_i^{(0)} N_i + A_i^{(1)} N_i N_i + B_{ij} N_i N_j$	$B_{ij} = 0,\ A_i^{(0)} = 0,\ A_i^{(1)} = 0$
Innovation by new agent $W(N_i+1, N_j \mid N_i = 0, N_j) = M_{ij} N_j$	$M_{ij} = f(c, n)$
Spontaneous new agent $W(N_i+1 \mid N_i) = \phi_i$	$\phi_i = 0$
Imitation $W(N_i+1, N_j-1 \mid N_i, N_j) =$ $A_{ij}^{(0)} N_j + A_{ij}^{(1)} N_i N_j$	$A_{ij}^{(1)} = h(gr, (1-gr), o),\ A_{ij}^{(0)} = 0$
Innovation by existing agent $W(N_i+1, N_j \mid N_j) = A_{ij} N_j$	$A_{ij} = g(gr, c)$

probabilities with which this happens depend on the gross return of the technology and the connection matrix. The higher the gross return of a technology, the more capable that technology is of supporting the establishment of supplying technology (i.e. vertical expansion to include backward links). Note that this additional technology can be new (in the sense that this will be the first occurrence of this technology in the model), or it can be an existing technology. In both cases, we say the agent is innovating in its production process by adopting the new technology. Finally, imitation occurs when a technology that is seldom used is replaced by a technology with a higher gross return rate. The probability of this happening also depends on the occupancy rate, so that when many agents already possess a particular skill, the probability of additional introductions of that skill is reduced accordingly.

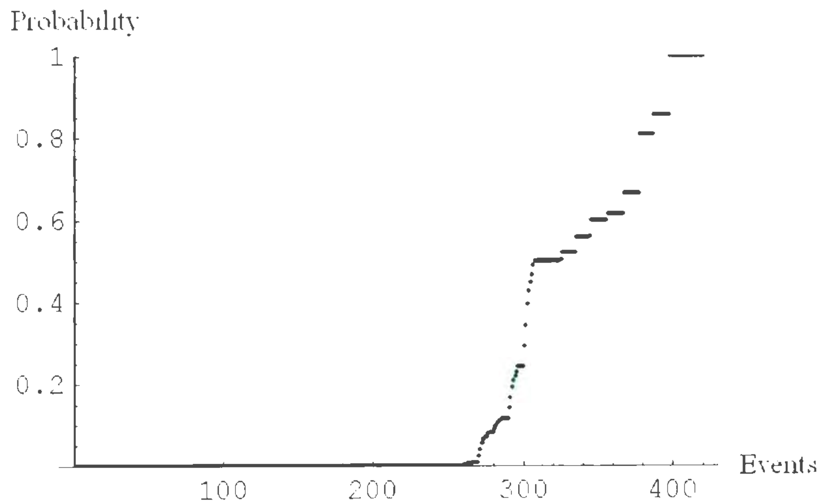


Figure 5.5: The cumulative probability distribution of imitation and innovation of the technologies.

For example, Figure 5.5 shows the cumulative probability distribution at the beginning of iteration 100. There are 10 skills, and thus $10^2 = 100$ instances of expansion through a new agent of a skill promoted by another skill, 100 instances of innovation by a new agent promoted by an existing skill, 10 instances of a spontaneous introduction of a new agent, 100 possible cases of imitation of a successful skill replacing another skill, and 100 possible cases of one skill encouraging the acquisition of an additional skill. Most of the probabilities for these 410 events are equal to zero, because the parameters are set equal to 0. Others are 0 because there is no new technology currently available, so there is no innovation by a new agent. Only the last two events, imitation and innovation by an existing agent, show positive coefficients. The coefficients are added and scaled in order to create a cumulative probability distribution, as in Figure 5.5. Since both types of events make up half the probability, imitation will occur with a random draw in $\{0, \frac{1}{2}\}$ and innovation by an existing agent will occur with a random draw in $\{\frac{1}{2}, 1\}$. If there is a new technology available, each of the types of events will have a probability of $\frac{1}{3}$.

After each iteration is finished, a new cumulative probability distribution is drawn up, a random number between 0 and 1 is generated, and the corresponding interaction between two technologies is selected. In the case of Figure 5.5, the event could be replacement of one skill by another, or the acquisition of a skill that will supply a technology. An agent that qualifies for such a change is selected at random, and the change in skills is made in the agent's skill list (see Figure 5.6).

```
imitation of skill 8 at cost of skill 5
{28, 29, 29, 16, 12, 19, 15, 15, 18, 29} changes to
{28, 29, 29, 16, 11, 19, 15, 16, 18, 29}

imitation of skill 9 at cost of skill 3
{28, 29, 29, 16, 11, 19, 15, 16, 18, 29}changes to
{28, 29, 28, 16, 11, 19, 15, 16, 19, 29}

imitation of skill 8 at cost of skill 2
{28, 29, 28, 16, 11, 19, 15, 16, 19, 29} changes to
{28, 28, 28, 16, 11, 19, 15, 17, 19, 29}

innovation within agent to acquire skill 6
{28, 28, 28, 16, 11, 19, 15, 17, 19, 29} changes to
{28, 28, 28, 16, 11, 20, 15, 17, 19, 29}
```

Figure 5.6: Logging of changes made to the distribution of skills. The reported changes are illustrated by changes to the total occurrences of technologies in the model, which has 29 producers and 29 consumers. Producer skills in high demand (7,8,9) are replacing less useful skills, or agents are adding skills that supply one or more of their other skills to their skill list.

5.2.3 Inventions and the adoption of new technology

Although the model shows interesting behaviour so far, and demonstrates that agents without a centralized control center can manage to feed and entertain themselves in a sustainable way, the real adventure lies in the introduction of new technology and products, and studying the effect of utterly new elements on the existing system. The framework developed in this thesis is set up in such a way that experiments of this kind are a straightforward expansion of the model above. The technology matrices are expanded at random times, as explained in Figure 4.1, resulting in a new column for a new technology, and new rows for new products. Based on the probability distribution described above, at some point an agent is assigned a new technology, and starts using the newly acquired technology if profitable. If the new technology does not suit the situation, because it does not improve profitability, the new technology might be short lived. Perhaps it would have done better in another location, but we may never find out.

New technology, if successful, will lead to new demands, i.e. innovation co-evolves with demand (Dawid, 2006), and new niches are created over and over again. New technology will compete with the other technologies for inputs, and change the flow of goods. This in turn will change the demand for skills. Technology once of paramount importance may be replaced by more efficient technology, or may fade because there is no longer demand for its output. New technology and demand for new products can wipe out what were once well greased production lines, and replace them with something that, for whatever reason, serves better, thus forcing the economic system to reorganize.

At this point in our simulation, a new product is introduced to the consumers. There are many possibilities for dealing with consumer preferences. One method is to set a exogenously fixed rate c of consumption, as was described in Section 3.3. Because of this minimum level of consumption, all consumers must acquire a certain amount of consumables, equivalent to what can be transformed into c units of labour. Those consumables are removed from the system (actually, the consumables are transformed into labour and these c units of labour are removed from the system). If there is a surplus of labour, prices for labour are such that consumption is not profitable. Nevertheless, the requirement for consumption c has to be met. As a result, with excess labour supply the consumers will consume precisely the required level and nothing more. If there is a demand for labour greater than the supply, generating labour becomes profitable, and consumers will consume as much as possible, while satisfying all constraints that apply. More advanced products will allow a larger output of labour. The total level of activity per agent is fixed and limited. However, with more advanced input, a higher output is generated. This leads to a preference for more advanced products, but only when demand for labour is high. When demand for labour is low, all the consumer must do is meet the consumption requirements. Therefore, the need for improved technology is not consistent, unless the forced level of consumption c increases over time, which is not entirely unrealistic. The difficulty lies in setting the correct rate of increase. When the increase is too low, there is no desire for improved technology, and when high capacity consumables are used, they are so productive in terms of generating labour, that it floods the whole system with an abundance that leads to temporary inactivity. When the increase is too high, the system grinds to a halt, since too many resources have been spent on discretionary consumption. After much cumbersome experimentation with appropriate increases, a different route was chosen.

A consumer agent's behaviour rules are such that they will optimize the quantity of labour generated within the usual boundaries given by the constraints, see Appendix B.4 for more details on the implementation. At the beginning of the turn, the agent considers his monetary resources, which possibly have increased due to the sale of labour or payments of dividend. The agent subsequently acquires those consumables that are available and that will allow it to generate as much labour as possible. When there is only one consumable available, as has been the case so far, there is only one option. When there is more than one consumable, consumers have to choose between the options based on their consumer preferences. Since consumer preference falls outside the scope of this thesis, the consumer preference model is kept as simple as possible, and the desirability of the consumable is given by the labour output coefficient. In Table 5.3, for example, both consumables, 104 and 27040, are equally desirable, since both generate 3 units of labour per 1 unit of consumer activity.

Table 5.3: Introduction of a new consumable in the system. Technology seven (column 7 of the input and output matrices in between the bars) shows the production process of a new consumable 27040. The last column shows how this product can be transformed into labour.

products	*input matrix*												*output matrix*											
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	**3**	**3**
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	$\frac{1}{5}$	0	0	0	0
110	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	$\frac{1}{5}$	0	0	0
27040	0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{6}$	0	0	0	0	0	0	1	0	0	0	0	0
154	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	$\frac{1}{5}$	0	0
130	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0
260	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	3	0	0	0
104	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	4	0	0

The output coefficient of a new consumable is a random number varying between the maximum labour output coefficient and double that coefficient. For the expansion illustrated in Table 5.3, a random number between 3 and 6 was generated. To maintain a constant return rate for all columns, the input coefficient was adapted such that the value of the output is twice the value of the input. Here, the input coefficient of the transformation of product 27040 into labour is equal to $\frac{1}{6}$. The reason for such a regulated output coefficient is twofold. First, a new consumable with a labour output coefficient below already existing output coefficients is a waste of simulation efforts, since it will not be incorporated. Even at the time of introduction, there already exist more desirable alternatives. Secondly, a new consumable with simply 1 unit of consumable as input coefficient and a calculated output coefficient (such that return rates remain equal) leads to an unrealistically rapid desire for change with this model of consumer behaviour. Based on one unit of input, the new consumable above would have an output coefficient of 18. Due to the chosen consumer preference model, this would result in a consumable far superior to the old one. Regardless of the implementation, ultimately the consumers move to the product that provides most satisfaction, but with a regulated and more gradual growth of output coefficients, consumer behaviour changes much more slowly and more realistically.

Figure 5.7 shows the adoption of the new technology to generate the new consumable 27040 by producers. Since the probability of imitating technology depends on the gross return of the technology, its imitation by numerous producers indicates that this technology is doing fairly well in comparison with the other skills, most likely because there is a demand for it, yet only a few providers.

Even though this new technology proves to be quite successful at the time of

Table 5.4: An expanding von Neumann input matrix. The columns between bars are new to the table in comparison with Table 5.3. In addition to two new consumables (1456 and 2600) and their consumption (t18 and t19) there are also two new technologies (t20 and t21) to help in the production of 2600. The latter two new technologies require three new products, t20 only needs capital product 220, t21 requires capital product 22 and input product 946400.

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	t19	t20	t21
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	1	0	0	0	1	0	0	2	1	0	0	0	0	0	0	2	0
7	0	0	0	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0
70	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
110	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
27040	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{6}$	0	0	0	0
1456	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	$\frac{5}{18}$	0	0	0
2600	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	0	0
220	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
946400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
154	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
130	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
260	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
104	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	1	0

Table 5.5: The expanded von Neumann output matrix matching the input matrix in Table 5.4.

	t1	t2	t3	t4	t5	t6	t7	t8	t9	t10	t11	t12	t13	t14	t15	t16	t17	t18	t19	t20	t21
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	3	5	10	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	1	0	0	0	0	0	0	0	0	$\frac{4}{5}$	0	0	0	0	0	0	0	0
110	0	0	0	0	1	0	0	0	0	0	0	0	0	$\frac{4}{5}$	0	0	0	0	0	0	0
27040	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1456	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
2600	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	$\frac{109}{50}$	$\frac{151}{25}$
220	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	$\frac{4}{5}$	0
22	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	$\frac{4}{5}$
946400	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
154	0	0	0	0	0	1	0	0	0	0	0	0	0	0	$\frac{4}{5}$	0	0	0	0	0	0
130	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0	0
260	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0
104	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0

```
innovation new agent with skill 7
innovation within agent to acquire skill 6
innovation within agent to acquire skill 8
...
imitation of skill 7 at cost of skill 1
...
imitation of skill 7 at cost of skill 2
...
imitation of skill 7 at cost of skill 1
...
imitation of skill 7 at cost of skill 4
```

Figure 5.7: Logging of the appearance of a new technology to produce the new consumable (product 27040). Adoption of the new technology shown here took place over 16 iterations. Due to the lack of other technologies that stand out in terms of their gross return, skill 7 quickly obtained a gross return sufficient to result in imitation probabilities high enough to cause repeated imitation events.

its introduction, the success is short lived. The quick exit of the consumable 27040 occurs not because there are problems on the supply side, but because demand fails due to the introduction of newer consumables and newer technology. This time, the newer consumables come with clearly higher labour output coefficients (see Table 5.4 and 5.5 where the labour output coefficients are 5 and 10 instead of 3), and as a result, they are more desirable to the group of consumers. Demand for the newer consumables is strong, and with the development of an industry capable of supplying these newer consumables, the older consumables fade. (see Figure 5.8).

The market share of each of the consumables is graphed in the left figure. It shows that while initially there exists only one consumable that fully controls the market, slowly competition appears, ultimately resulting in four consumables with different shares in the generation of labour. Skill 17, using consumable 27040, is initially quite successful, and quickly captures roughly 25% of the market. However, with the introduction of more favourable consumption goods, it quickly looses the ground gained. The rising star is product 2600, which, shortly after its introduction,

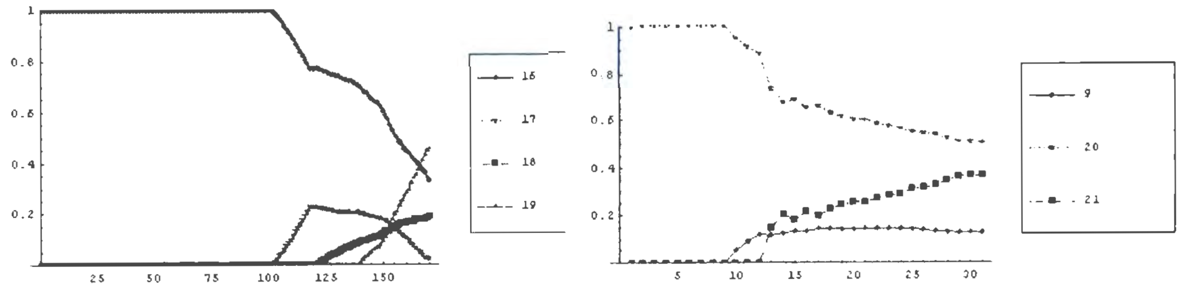


Figure 5.8: Both graphs display the market shares of technologies producing the same product. The graph on the left shows the share of four skills (each using a different consumable) in the generation of labour. The graph on the right shows that the consumable that was introduced last (2600), and that has only been around for thirty or so iterations, is already produced by three different technologies, each of these competing to meet the demand for product 2600.

is already responsible for most of the labour generated. Initially, only technology 9 produces 2600. The right graph in Figure 5.8 shows how newly introduced technology is joining technology 9 in order to produce enough of product 2600 to satisfy the increasing demand.

Already, with the technology set in Tables 5.4 and 5.5, interesting, realistic behaviour is obtained. We see persisting asymmetric performances in sectors, we see life cycles of products, and we see instability in market shares. The life cycle of product 27040 was just discussed. Although the product experienced a brief period of success, it quickly becomes obsolete due to the introduction of better and more desirable products. The technologies responsible for the production of product 2600 each have different input and output coefficients, although for the production of $2600 = 2^3 \cdot 5^2 \cdot 13$, the products 2,2,2,5,5 and 13 are required, regardless of the production technique. Due to the use of different inputs, different production techniques can coexist, and display quite unstable market share behaviour. See for example Figure 5.9, which shows the market share of 5 technologies all producing product 2600. It shows that these different techniques coexist for quite a long time, doing better or worse now and then, because they have to compete for their inputs with other technologies. If competition for one input is fierce, production based on that input will possibly be lower and a competitor using different inputs will benefit. When access to the input improves, the technology can regain its former market share.

Finally, the last graph in Figure 5.10 shows the market share of the different consumables in the production of labour. To be more precise, the graph shows the market share of 11 different skills used to generate labour, but each of these skills uses a unique consumable, and thus one can speak of the market share of the consumables, instead of the market share of the technologies. As the figure shows, with the introduction of a new consumable, the old consumables are doomed to become obsolete. One by one, the consumables are replaced by more advanced products that result in higher labour output. However, the replacement is not instant due to the sometimes small differences in desirability, or due to difficulties encountered in establishing a production line for the new consumable capable of supplying the whole market.

In reality, new consumables are often added to the existing gamut of consumables, without replacing all of the previous products. In that respect, the simple model of consumer preferences is not very realistic. However, when the whole model discussed here makes up one sector, for example computer electronics or sound systems, then new consumables do replace older goods. In addition, although it is an essential part of modelling a market, this thesis does not deal with consumer behaviour specifically. However, the consumer preference model used here does suffice

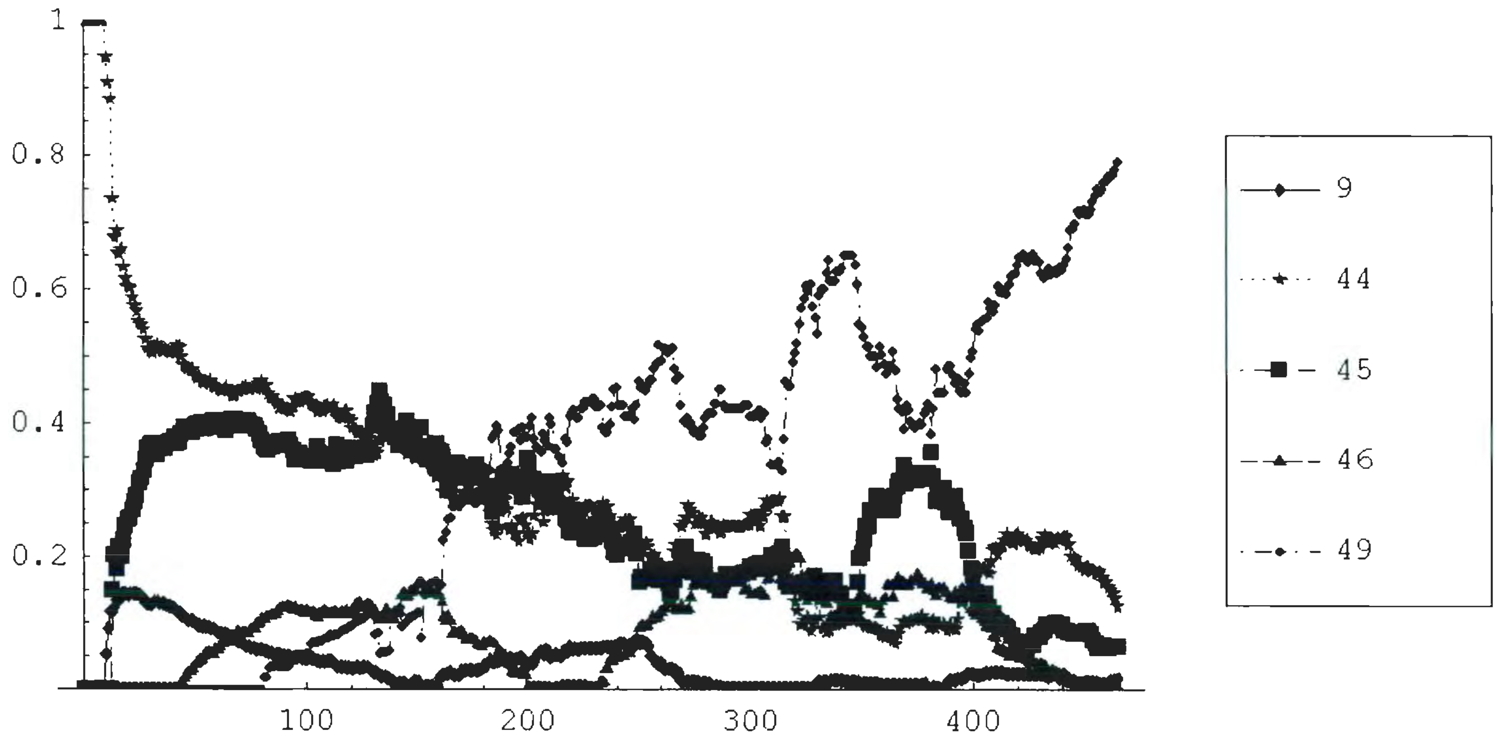


Figure 5.9: Market share of 5 different production techniques used to produce product 2600.

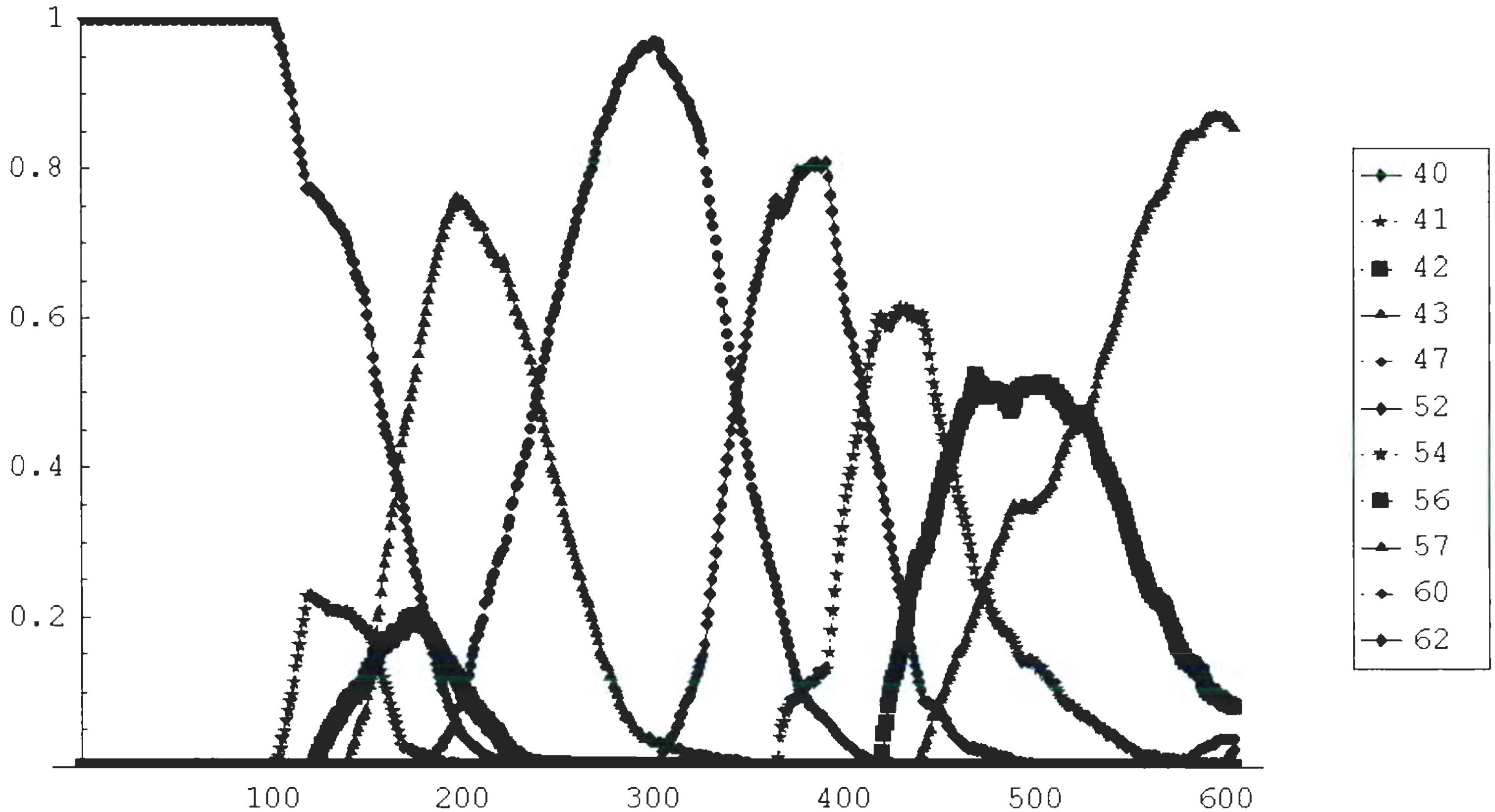


Figure 5.10: Market share of 11 different production techniques involved in the process of generating labour.

in creating a explicit complete market model in which there is demand for newly introduced products. The market model, in itself, is not the focus of this thesis, but is included in order to select from the the variation that is introduced into the model by innovation and diversification. As such, the market model does what it is supposed to do, and this chapter has attempted to give a first indication that this is the case. In the next chapter, the model results are analysed in much more detail.

Chapter 6

Results and analysis

The previous chapter showed how a typical model is set up, what a typical model run looks like and what typical output is generated. This chapter will treat the output in more detail and will extract and analyse a number of interesting phenomena generated by the model and compare these with reality.

This chapter continues the discussion of market shares that was started in the previous chapter. In contrast with the model described in the previous chapter, the consumers in this chapter are obliged to consume an exogenously determined rate of consumables. This rate is fixed and does not increase during the simulation. As new products and consumables are introduced, the labour output coefficient of the consumable does not affect consumer preference, the consumer's only requirement is to satisfy the obliged consumption, equivalent to c units of labour. Which products it will use to do so will depend on the availability and price. Related to this different approach in the model is the method to determine the labour output coefficient of a new consumable: here simply the general rule that applies to all production techniques is applied, being that one unit of input results in the general profit rate. There is no manipulation of the new output coefficient, some output coefficients will be lower than those already existing, some will be higher. As discussed in the previous chapter, such model of consumer behaviour does not encourage growth. However, such a model does illustrate the capability of the model to adapt and to reorganize itself by redirecting flows of goods and money in order to create a better fit between supply and demand.

Secondly, the chapter returns to the stable Leontief activity levels. Where in a previous illustration the resulting production numbers were compared to the production numbers one could expect based on the Leontief analysis, this section will explore the capacity of the agents and the auctioneer to organize the production system.

Section 6.3 studies the methods available to determine the presence of sets of technologies that can function independent of the other technologies, so called functionally closed sets. The interest for these functionally closed sets lies in the fact that they provide insight into what is required for a well functioning system. They give information on which combinations of functions, qualitatively and quantitatively, have the ability to perform well. Such clusters of technologies are good candidates for establishing independent organizations that are self supporting.

Finally, section 6.4 applies a measure developed by Bedau, Snyder and Packard to analyse the class of evolution the model is capable of generating. The different approaches applied in this chapter illustrate the wide reach the model displays. The first section is mainly descriptive, the second section falls back on economic analysis. The third section uses concepts borrowed from artificial chemistry concepts and techniques derived from graph theory. The last approach of analysis stems from biology. The model has been viewed from several different perspectives.

6.1 Market shares continued

The following results are a description of model output after 25 events in which new technology or new products were introduced. Some of these were successful, at least for a while, some never made any change to the system. The invention events occur at random intervals, and are separated by periods of economic activity during which the set of products and technology is not expanding. All together there were close to 3000 iterations, in each of which every agent took part in the economic process: producing, trading, building a network. The average number of agents was about 40, and this leads to about 120000 agents' actions in an economy with 41 products and 52 production technologies after clean-up. Clean-up here consists of removing the technologies which none of the agents is capable of using (the ones that were capable have gone bankrupt or exchanged the skill for another more useful one) and removing the products that no longer play a role after the removal of the unused technologies.

First we show in Figure 6.1 the market share of each of the consumables. Ultimately there were 8 products that could be consumed (and thus transformed into labour). However, most of them never really were successful. One can see a brief life of a product around iteration 1500 on the x-axis as it almost reaches 20% of the marketshare, but shortly after that it pretty much disappears as it dives below 5% marketshare, as many of the other new consumables do. However, the consumable associated with transformation by technique 35 turns out to be very succesful and more or less replaces the original consumable that the model started with. Here we see an excelent example of a constructive system at work: a new product composed of primitive products finds itself somehow in a situation (better supply of input, better location with respect to the market, immitation behaviour of agents copying succesful production) in which it can grow and finally dominate the market. Moreover, the model itself introduces the new technology and the new products based on endogenous factors, the modeller has nothing to do with innovations. It is also the model itself that decides whether or not a new process is going to be succesful, and this is achieved through the application of the market mechanism.

The replacement of one dominant consumable by another obviously changes the production system. The production of 140 is on the rise as the production of 104 is decreasing. These two products are contesting the domination of the consumer market. As product 140 is gaining market share, the production of inputs required for it is increasing. As 104 is losing, the production of its inputs is decreasing. Figure 6.2 shows the rise and fall of the two sectors.

However, within one sector also competition arises. Initially product 140 was produced by combining 2 and 70. Soon, however, another production technique was introduced to make product 140 out of new product 18200 and also out of new product 342000. See Figure 6.3 for the competition for the input market of product 140. The first producer of product 140 by means of technique 7 is completely replaced by new techniques 39 and 40. After a battle for the market, technique 39 wins.

In other simulations we see similar results. A new consumable (sometimes) takes over the market, and therefore the production of the old consumable wanes. Naturally, the input required for the old commodity dwindles too. In Figure 6.4, product 104 was produced out of labour and product 260, and 260 was produced out of 130. It is interesting to see that while the production of 260 more or less falls

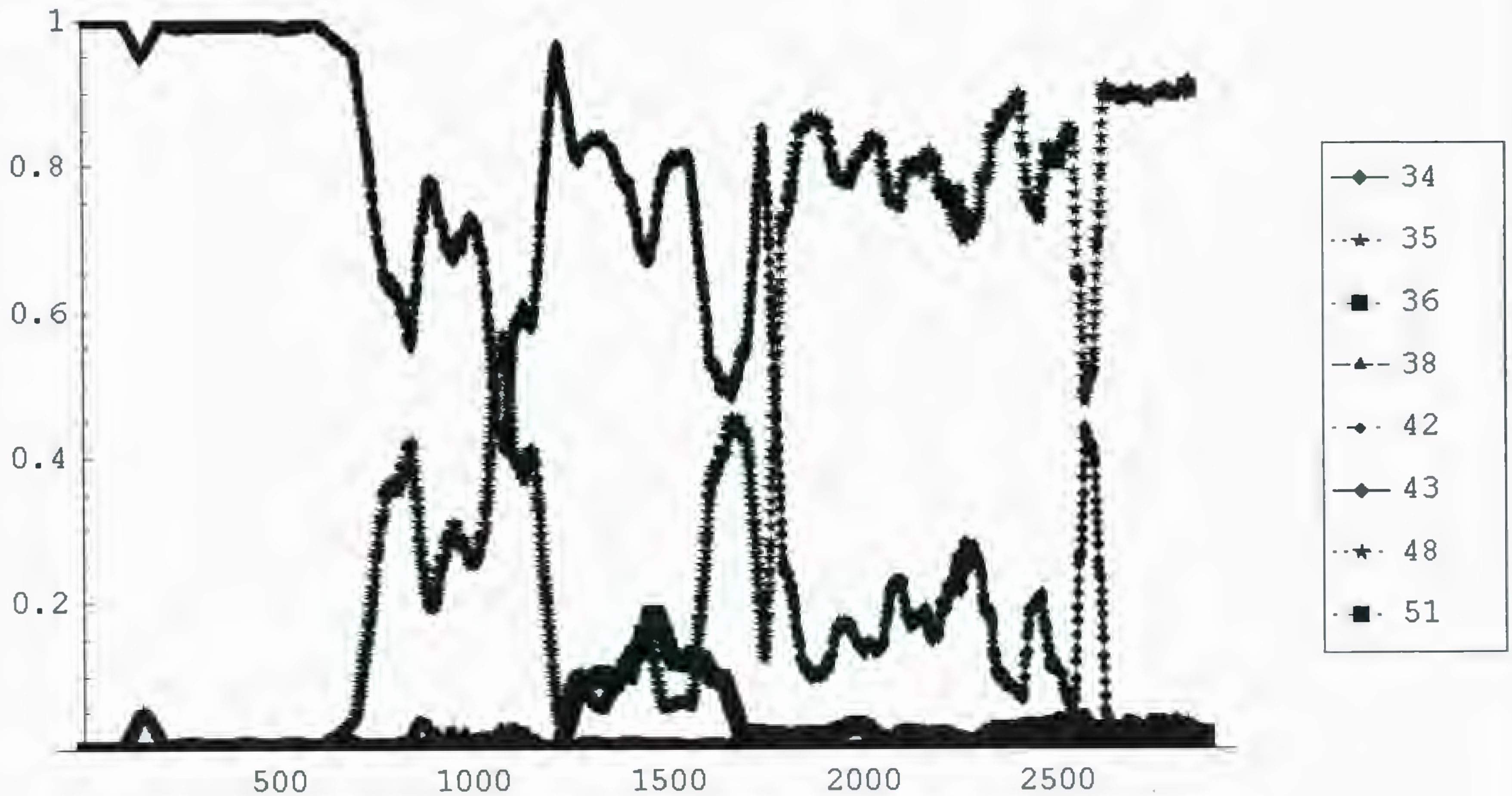


Figure 6.1: Market share of 8 different production techniques calculated based on average production figures.

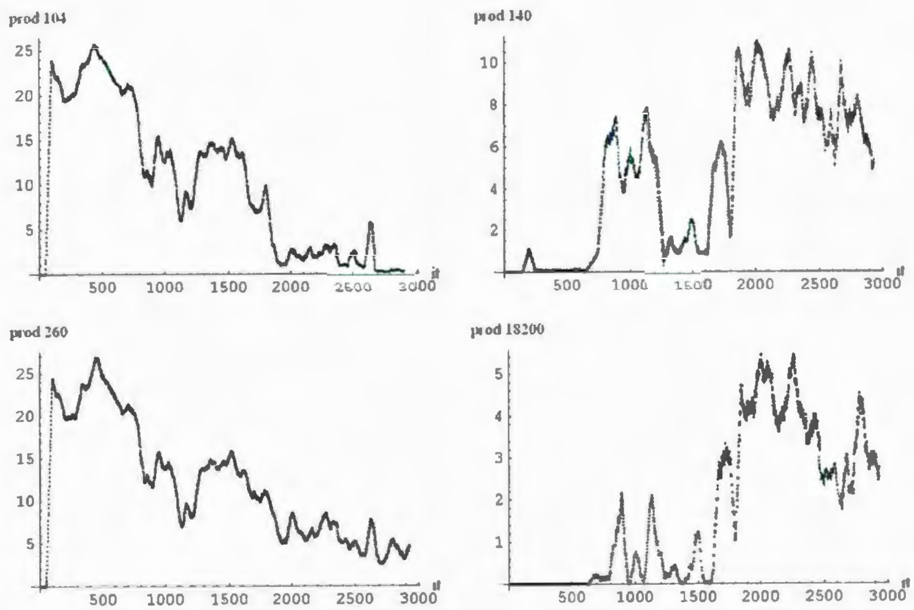


Figure 6.2: Production of old and new consumable. The left column shows the declining production of product 104 and its input 260. The right column shows the production numbers of the new consumable that replaced the old one and its main input (product 18200).

with the production of 104, the production of 130 remains strong, simply because that product has become the input for another product that is doing well. Figure 6.4 illustrates the occurrence of changing flows between sectors in the model, and shows that the artificial economy has to reorganise itself when technology changes.

These few examples illustrate the possibilities of this model. It allows one to simulate an innovative economy, in which products play their part until they are replaced by other products that are more profitable to produce. There are various reasons that can make a product or production technique more suitable: better access to input, higher output per single execution, better located with respect to the market and so on.

This section concludes with a visual comparison of two images of market share dynamics in Figure 6.5. The first image displays the competition between production

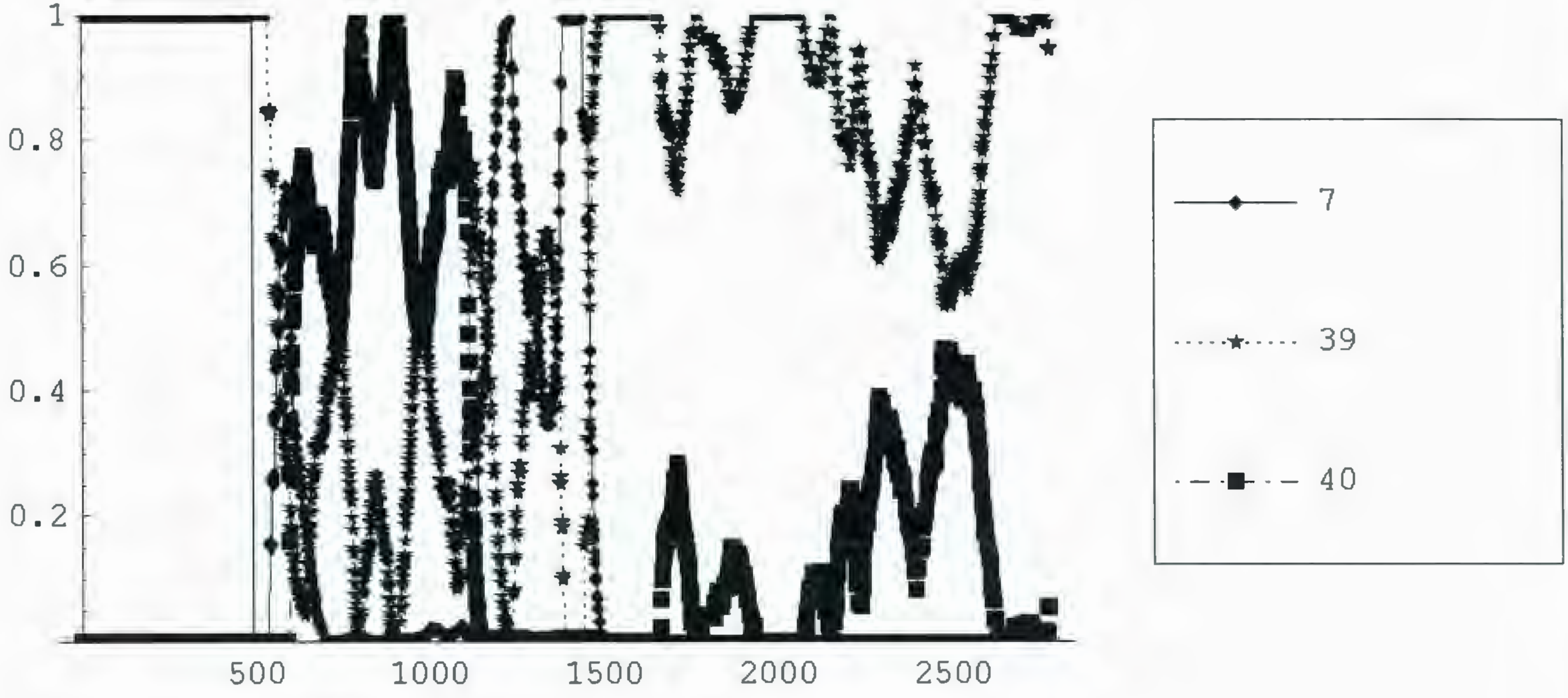


Figure 6.3: Market share of 3 different production techniques for producing product 140. The technique that introduced the then new product 140 is quickly replaced by two techniques with a higher capacity, of which technique 39 (column 39 in the input and output matrices) ultimately dominates the market.

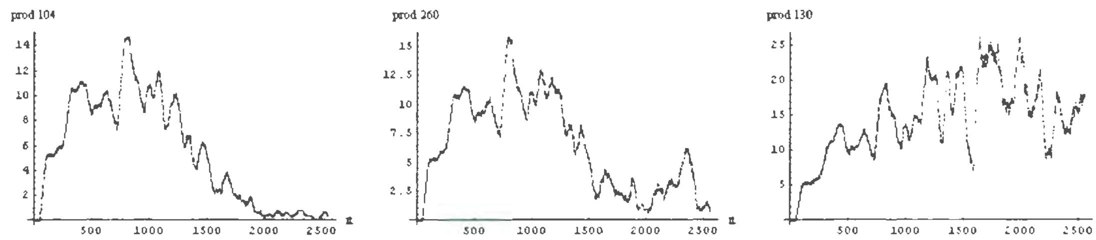
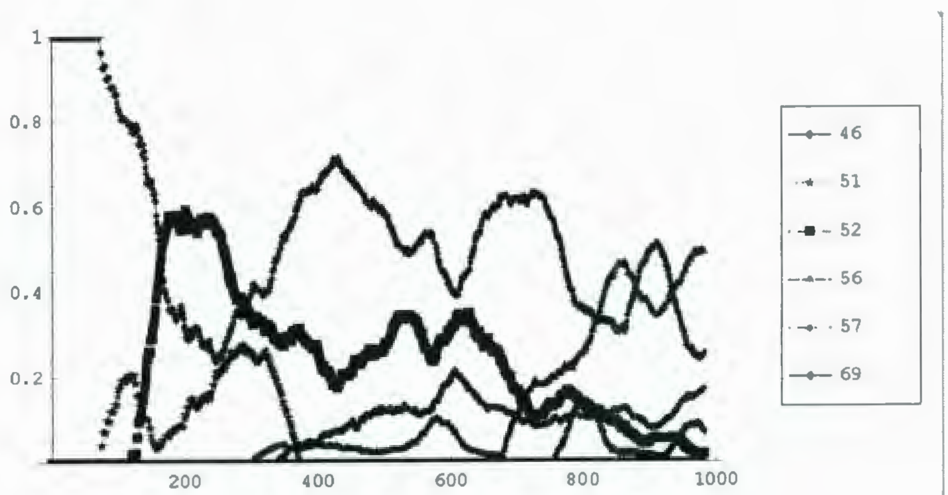


Figure 6.4: Production of consumable 104. As consumable 104 is replaced by a new product (left), the demand for the input 260 for producing 104 disappears too (middle). However, product 130 which was formerly involved in the production of product 260 obtains a new role and its production is continued (right).

techniques for product 104, similar to the market share graphs we have seen above. Again, existing technology is partly replaced by new technology. The first newly developed technology that conquers a substantial part of the market is technology 52, represented by column 52 in the von Neumann input and output matrices. As can be seen, this technology disappears toward the end of the simulation run, illustrating a life cycle of the involved products. The second image shows the market share of energy sources in Canada between the years 1871 and 1996. The model of this thesis which was used to generate the results of the top image is completely abstract, and does not simulate the use of different sources to generate energy in Canada. Nevertheless, the resemblance of the instabilities in both graphs is striking. The figures are included in this thesis to illustrate how realistic the competition for market share resulting from the model possibly is. Further research on market share results is required.



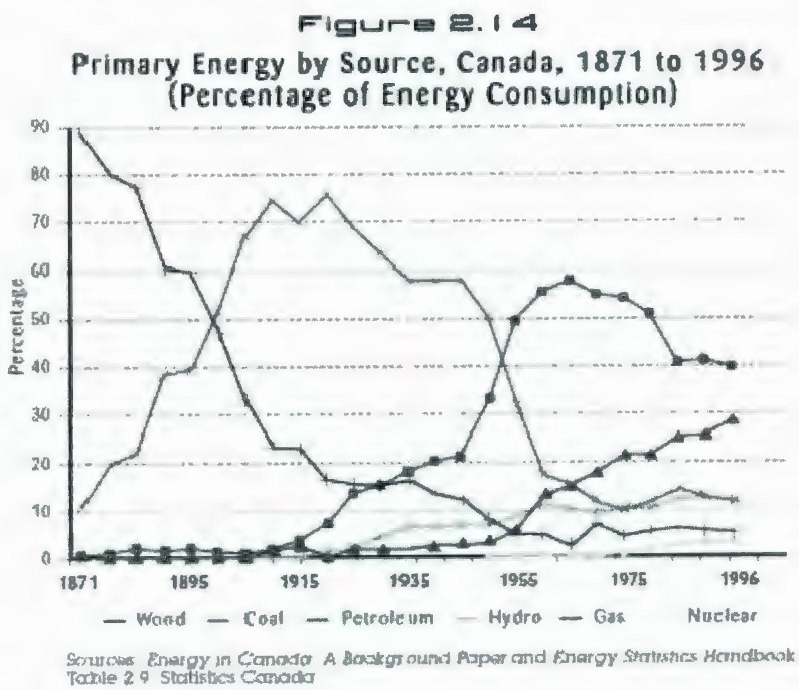


Figure 6.5: Visual comparison of market share dynamics. The top graph shows the market share of 6 technologies used to produce product 104. The numbers on the right indicate the number of the column in the pair of input and output matrices. The bottom graph is borrowed from Statistics Canada and displays the raw materials used to generate power in Canada.

6.2 Stable Leontief activity levels

In a Leontief system there are as many production processes as products. When our model is set up to represent such a system, straightforward linear algebra allows us to calculate the activity or intensity per iteration that is required to supply every agent with the input it needs. In such a system, the production per iteration will converge towards the Leontief stable equilibrium, regardless of the initial situation. This occurs even though the agent's environment consists only of the prices, its own stocks and the stocks of its trading partners. The agent has no direct information on the "required" activity per iteration, but it does receive signals through the prices set by the price mechanism. The price mechanism uses the production numbers over the last w iteration(s) and attempts to maintain in stock as much as is required for these actions, as was explained in Chapter 3. Through this price mechanism, an appropriate production is encouraged, and an attempt is made by the agents to keep in stock what was used in the last w iterations, where w is a parameter specifying the length of the auctioneer's memory. This leads to the Leontief stable state, as was illustrated in Figure 3.3. The production per iteration level converges towards the Leontief stable state.

In this section a similar experiment is described, but here, the capacity of the agents and the auctioneer to organize the Leontief production system is explored. By repeatedly increasing the forced consumption rate, agents are seduced by opportunities to generate increased profits, and they adapt their production efforts to the new situation. By increasing the forced consumption rate of consumers toward the maximal sustained rate, one can obtain an idea of how well the price mechanism functions. When the forced consumption is at such a level that the agents have to function at their maximum capacity, there is no room for mistakes, and the prices have to instruct the agents in a timely manner and with precision.

For this experiment, the technology used is slightly different from the one used in Chapter 3. The products are the same and the transformation of products into more advanced products is the same, but the output coefficients are different. For simplicity, the output coefficients for all products other than labour are 1, which makes the activity level of the technologies equal to the production. The return rate for each of the technologies is equal to two. This results in the set of technology matrices in Table 6.1. The corresponding price set is given by $\{1, 1, 2, 2, 2, 0, 6, 14, 30\}$, where labour is the numeraire good. The reader can verify that with the price of labour equal to one, and a return rate equal to two, the prices of the raw materials 5,7, and 11 are $2(1) = 2$, the price of product 130 is $2(1 + 2 + 0) = 6$, the price of product 260 is $2(1 + 6) = 14$, and the price of product 104 is $2(1 + 14) = 30$. Thus, the output coefficient of labour must be equal to 60 if the return rate of the activity in the last columns is equal to those of the other columns, namely 2.

Agents are randomly distributed on a 14 by 14 cell grid, resulting in 39 agents. All agents possess skill 1, enabling them to generate raw material 5 out of labour. Then every agent randomly selects one of the skills 4,5 or 6. A consumer agent is placed with each of these producer agents, and thus, every occupied location has a producer and a consumer. In total the system consists of 78 agents. The total presence of skills in this system is $\{39, 0, 0, 12, 14, 13, 39\}$. Since every agent's total activity per turn is limited to 4, which is a model parameter, the total activity per iteration must be less than $\{156, 0, 0, 48, 56, 52, 156\}$. To allow the maximum production capacity to be possible, all agents are connected to all other agents, such

Table 6.1: The possible transformations, where three of the output coefficients are set to 1 and the final output coefficient is determined such that all sectors have an equal profit rate of 100%.

products	*input matrix*							*output matrix*						
2	1	1	1	1	1	1	0	0	0	0	0	0	0	60
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	0	0
13	0	0	0	1	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	1	0	0	0	0	0	1	0	0	0
260	0	0	0	0	0	1	0	0	0	0	0	1	0	0
104	0	0	0	0	0	0	1	0	0	0	0	0	1	0

that a local shortage of resources cannot hold back the agents.

The resulting activity vector z has to be such that the outcome is sufficient to cover the input plus the forced consumption. Forced consumption in this model is given by a quantity of labour c, the result of the transformation of consumables and removed from the system for each of the consumers. Suppose each of the consumers produces labour whenever there is demand for it, but regardless of that demand each consumer also acquires enough consumables to generate $c = 10$ units of labour. For 39 consumers, this implies that the forced consumption demand vector D equals $\{390, 0, 0, 0, 0, 0, 0, 0, 0\}$ and when A is the input matrix and B the output matrix

$$B \cdot z = A \cdot z + D \Leftrightarrow (B - A) \cdot z = D \Leftrightarrow z = (B - A)^{-1} \cdot D$$

which is the same equation as was used in Section 3.3. For D as above this results in $z \approx \{6.96, 0, 0, 6.96, 6.96, 6.96, 6.96\}$. It is easy to see that when forced consumption is about seven times as high as in the above example ($c = 70$), the required activity vector is seven times z, or roughly $\{48.75, 0., 0., 48.75, 48.75, 48.75, 48.75\}$. This becomes a problem since there are only 12 agents with technology 4: their maximum production level per iteration is 48, which is only possible if they can completely devote themselves to this task and if they always have enough resources to do so.

The experiment below shows how close the forced consumption can be brought to the maximum rate of $\frac{48}{6.69} * 10 \approx 69$. The simulation starts with sparsely stocked agents and with forced consumption at a moderate level of $c = 10$ units of labour per consumer. Once the initial shock of adapting to this level of forced consumption is over, and the agents have adapted to such activity levels and have acquired the corresponding stocks (remember that appropriate stock levels depend on the history of activity, and thus when activity levels change, stock levels change), the forced consumption level is increased by 10 units. This results in a simulation with the forced consumption level c being increased from 10 to 20 to 30 to 40 to 50. Then it is increased to 55 and finally to 60. The results are presented in Figure 6.6.

The changes in forced consumption are clearly visible in the production of labour, which shows a distinct increase corresponding to each change in forced consumption. The growth in labour production leads to an increasing demand for consumables. Indeed, product 104 also shows fairly clear steps with every increase. The increase in demand cascades down to the next product, 260, then to product 130, and finally, to product 5. In order to better see these trends, the right column of Figure 6.6 again shows the same data but now averaged over six iterations. More iterations would give a smoother result, but would also average out the initial steps leading to the forced consumption equal to 50, 55, and finally 60. When the forced consumption is equal to 60, still well below the calculated upper boundary of 69, the resulting production figures become rather chaotic. It may be that the price mechanism is

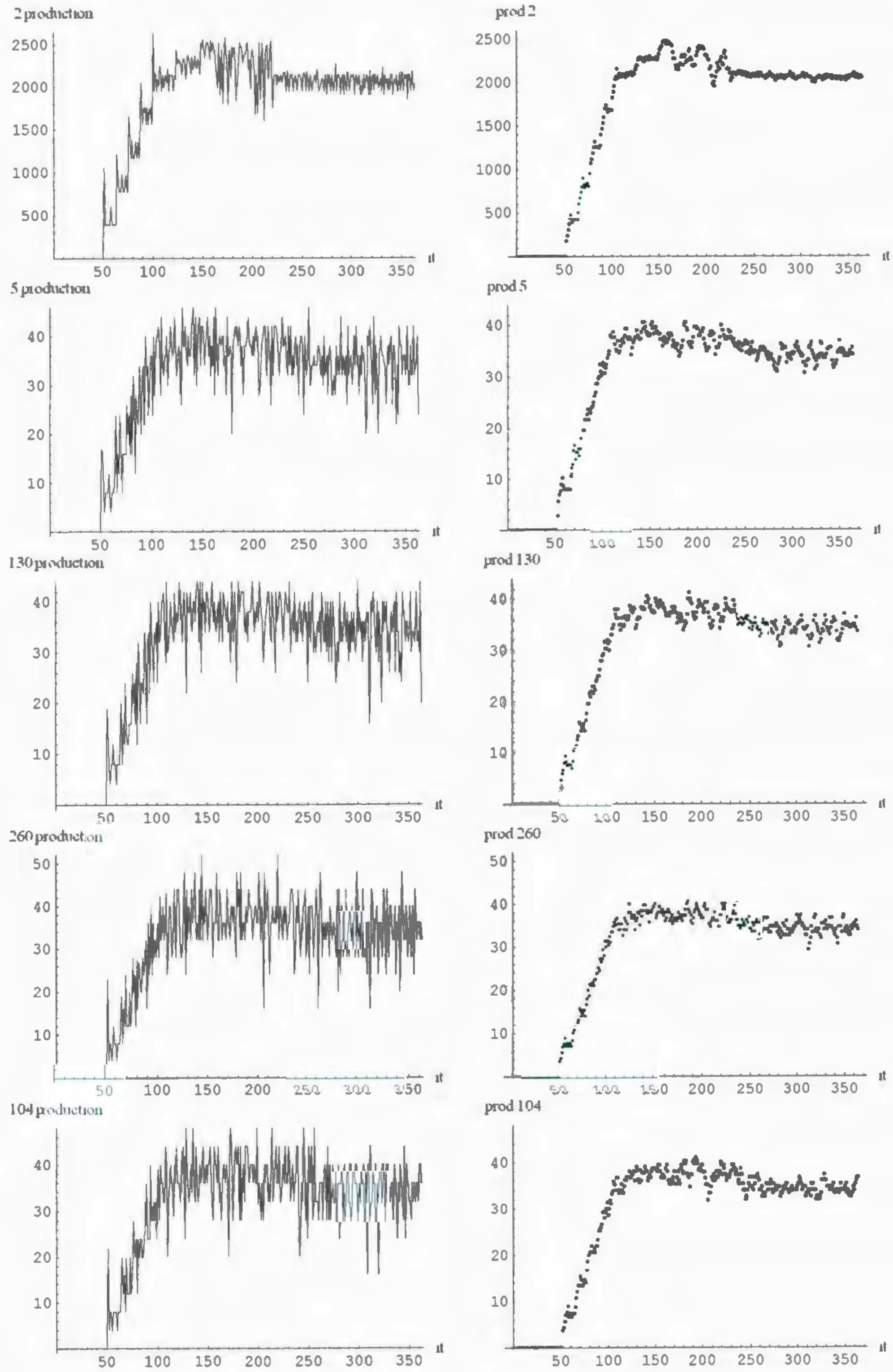


Figure 6.6: Stable behaviour corresponding to various forced consumption levels. The left column shows the raw production numbers per iteration, while the right column displays the same data but now as a gliding average over 6 iterations.

not precise enough to regulate the more critical production rates, or that it is too slow due to the long history of w iterations used in calculating prices. However, other model features can also contribute to the inability of the agents to generate this level of goods. When almost all agents possessing skill 4 are required to apply this skill to the maximum allowed level, they need to be able to do so, which means that they need access to the resources required to carry out this skill, in this case product 13, which is free and abundant, and product 5. Every agent possesses skill 1 to generate product 5; however, when an agent needs to execute both skill 1 and skill 4, it cannot execute skill 4 the required 4 times, since some of the total activity per turn has to be used on skill 1. Furthermore, due to the random ordering of agents per iteration, it is possible that agents are put in the "wrong" order. For example, too many agents with skill 5 may get their turn before their suppliers, agents with skill 4, get their turn. If there is sufficient stock of the output of skill 4 (product 130) then this should not be a problem. However, when the system is stretched, the random order can certainly become a factor.

As is shown in Figure 6.6, from iteration 150 to iteration 220 there is no regularity in the production numbers. Only after iteration 220, when the forced consumption is reduced to 50 units of labour per consumer, does the production return to a stable state. The gliding average with a window of 6 iterations long does not smooth the results quite as much as in Figure 3.3, but in that figure the window was 50 iterations long. Nevertheless, one can observe that the production levels hover around the 35 level, which is the number of each of the products (except labour) required to support a forced consumption of 50.

When there are different production processes for the same product (input substitution) as is generally the case in von Neumann technology models, it is no longer obvious which of the activities will produce which products. In advance it is unclear which combination of skills the agents will use; during a simulation the model itself will engage the appropriate activities to meet the demand. There are algorithms to calculate the possible combinations of technology that will guarantee a sustainable production system as the next section will explain.

6.3 Autocatalytic sets

Chapter 2 mentions a method for determining the presence of autocatalytic sets in an artificial chemistry (Jain and Krishna, 1999). Such sets are of interest here as well, but for two reasons the situation is different from that described by Jain and Krishna. First of all, instead of the focus on catalysts, which are assumed determinant in artificial chemistry, all of the inputs of the production process have to be included. Because capital, equivalent to catalysts in chemistry, is reusable, agents make money mostly by providing inputs other than capital, although some will specialize in building capital such as machinery. Capital has to be present in a transformation, along with the other inputs required. However, the capital is still present (apart from a limited level of wear) after the production process finishes, while the other inputs are used up. The economy runs on the provision of inputs for each of the processes. Instead of autocatalytic sets, the focus here is on functionally closed sets, i.e. sets of technology which provide all of the required input for each of the technologies in the set. Secondly, because the commodity population dynamics is based on an artificial market, and not on the abstract population model used by Jain and Krishna, the dominant autocatalytic set as defined in their paper does not suffice. This implies that different techniques to determine the functionally closed sets are needed.

The interest for these functionally closed sets lies in the fact that they provide insight into what is required for a well functioning system. They give information on which combinations of functions, qualitatively and quantitatively, have the ability to perform well. Such clusters of technologies are good candidates for establishing independent organizations that are self supporting. For reasons explained below, there has not yet been a thorough investigation into the appearance of such sets in the model, but such work certainly has potential: it indicates which groups of technologies should be placed together, and in what proportions. If we could construct a factory wall around such a group, which could be achieved in the model by placing the group of skills in one agent, then the independently producing system no longer has to hope that suppliers will be available, and surplus of production is also at their own disposal (i.e. Eigen's compartmented hypercycle (Eigen, 1992)).

In reality, companies are never totally independent of the outside world: most companies require labour, energy and raw materials from other suppliers. However, there are many examples of businesses expanding vertically to smooth interactions with suppliers or buyers, in an attempt to reduce outside dependence and to keep the resulting surplus within the organisation.

Also, these observations suggest that there is a certain analogy with biological systems or with Kauffman's autonomous agents (see Section 2.3), which are catalytically closed sets capable of performing a work-cycle. The observations may also suggest that there is a certain analogy in the organisation of such systems. The organisation of smaller clusters of functions is manageable, but when the system becomes too complex, the direction of the whole system has to be subdivided and a hierarchical organisation is often the result. A cell in a human body is already a very complex system: it requires raw materials from the outside, but it more or less functions on its own. However, there is no central management center in the cell either: the cell consists of many smaller bodies and apparatuses such as lysomes, microbodies, mitochondria, golgi apparatuses etc. (Carola *et al.*, 1992), that also work more or less independently of their surrounding, except for certain inputs. Whether or not such kinds of hierarchical organisation applies to cells, organisms, or companies is a question that goes far beyond the scope of this thesis. However, it is a very interesting idea, and the method described here of identifying functionally closed sets will eventually help in researching such questions.

6.3.1 Functionally closed sets

Table 6.2: Leontief technology system

products	*input matrix*							*output matrix*						
2	1	1	1	1	1	1	0	0	0	0	0	0	0	6
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	1	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	1	0	0	0	0
13	0	0	0	1	0	0	0	0	0	0	0	0	0	0
130	0	0	0	0	1	0	0	0	0	0	4	0	0	0
260	0	0	0	0	0	1	0	0	0	0	0	2	0	0
104	0	0	0	0	0	0	1	0	0	0	0	0	$\frac{7}{3}$	0

Suppose we have the set of production techniques of Table 6.2, which was also discussed in Chapter 3. Since this technology set is a Leontief dynamic system, we can treat it as a generalized eigenvector problem and calculate the Leontief stable state, which is the solution of $A \cdot \alpha z = B \cdot z \Leftrightarrow (A - \frac{1}{\alpha}B) \cdot z = 0$. This results in $z = \{6, 0, 0, 3, 6, 6, 7\}$, or any multiple hereof. This activity vector indicates the presence of a functionally closed set. The combination of these production processes, in the above quantities, results in a system that provides all of the required input for each of the processes in the set. Since this system consists of square matrices, z provides the unique solution, and thus it is the only case of a functionally closed set for this system. Suppose now that the above technology set is part of a larger set as in Table 6.3.

Table 6.3: Technology matrices

products	*input matrix*												*output matrix*											
2	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	6	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
110	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
154	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1
67600	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
130	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0
260	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	2	0	0	0
104	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	$\frac{7}{3}$	1	$\frac{22}{3}$

This set of technologies is not just larger than the above Leontief technology set. It also has multiple technologies to produce the same product (input substitution), and it features joint production. For example, the last column displays a technique with multiple outputs, and one of these is the same as the output of the third column from the right. The first difference is important here, since this implies that the matrices are not square, and thus the number of technologies m is larger than the number of products n. Therefore, the solution of $A \cdot \alpha z = B \cdot z$ cannot be determined by treating it as an eigenvector problem or by using the Leontief inverse matrix.

It has been shown that for n products there exists an optimal process which involves at most n techniques (Kemeny *et al.*, 1956; Gale, 1956). Weil uses this idea to develop an algorithm for the calculation of activity levels (Weil Jr., 1964). The algorithm consists of selecting n out of m processes which then can be treated as a generalized eigen problem. However, the number of combinations $\binom{n}{m}$ soon becomes very large, even when n and m increase only slightly.

Thompson (1974) develops an algorithm based on a two player fair matrix game. In a two player matrix game the $m \times n$ payoff matrix G represents the outcome of combinations of game options, where the m rows represent the m different options for player R and the n columns represent the options for player C. g_{ij} is the amount player C has to pay player R when player R plays option i and player C plays option j. In the case g_{ij} is negative, player R has to pay player C $-g_{ij}$. Of course, player C attempts to minimize the outcome of the game, while player R attempts to maximize the outcome. A pure strategy is an absolute choice for one of the rows or columns (such as $\{0, 0, 1, 0, \ldots, 0\}$), and a mixed strategy consists of a probability vector of length m for the player R and a probability vector of length n for player C. A probability vector is a vector with all elements greater than or equal to 0, and the sum of the elements equal to 1. A solution to the game G consists of the value of the game v, and strategies x and y, such that $xG \geq \{v, \ldots, v\}$ and $Gy \leq \{v, \ldots, v\}^t$ where the first vector of v's has n components, and the latter transposed vector of v's consists of m components. A game is called fair when $v = 0$ (Morgenstern and

Thompson, 1976). Kemeny *et al.* (1956) shows that von Neumann technology input matrix A and output matrix B can be regarded as game matrices, and that a von Neumann solution (i.e. an activity vector z and a price vector p) corresponds to optimal strategies x and y, such that when $M_\alpha = B - \alpha A$, $xM_\alpha \geq 0$, $M_\alpha y \leq 0$, and $xBy > 0$, where α is the expansion factor. In other words, find an α (there can be more α's) such that $v(M_\alpha) = 0$, and x and y are optimal strategies, such that the output generated by vector x and valued by vector y (i.e. xBy) is greater than 0.

Optimal strategies in matrix games can be determined by linear algebraic techniques, and thus optimal activity levels can be determined by linear programming techniques as well. Thompson provides an algorithm to determine the set of expansion factors and the corresponding activity and price vectors. If we look at the whole model, all technologies have the same return rate and expansion factor, which is fixed, and when new technology is added to the system, the same return rate is used to determine the input and output coefficients. Therefore, in this case there is no need to find the different α's. Part of the algorithm can be used to determine the extreme points of the convex solution space, and this is precisely what is needed in order to determine the set of functionally closed technology clusters in the model here.

Thompson regards the set of extreme points of the solution space as the vertices V of a graph $G = (V, E)$, and the edges E of the graph are defined by the following rule: an edge between two extreme points z_1 and z_2 belongs to G if it is possible to go from extreme point z_1 to z_2 by replacing exactly one basis vector by another basis vector. Through pivoting of tableaus, a standard technique applied to solve systems of equalities by means of the simplex method (Bazaraa *et al.*, 1993), it is easy to determine adjacent solutions. Subsequently, the algorithm is applied to search all nodes of the graph G to determine the set of extreme points.

The Thompson algorithm has been implemented here in Mathematica to determine the set of basic solutions to the following problem: For von Neumann technology input matrix A and output matrix B with n products and m technologies, and with fixed expansion factor α, find the solution of

$$A \cdot \alpha z = B \cdot z$$

where z is the vector of activity for each of the technologies. The solution space of this equation is a convex space, and by determining the basic vectors we can write any solution as a linear combination of these vectors. In particular, the basic vectors give combinations of technologies that are functionally closed. In other words, the set of technologies with activities greater than 0 in a solution z indicates the combination of technologies and the appropriate activity levels needed to generate all the required input for each of the technologies in this set. It is not just functionally closed; it provides enough resources to allow an expansion rate α such that even when activities increase with rate α, the combination of technologies is capable of maintaining a positive balance.

When we apply this algorithm to the matrices in Table 6.3, we get the following two functionally closed sets of technologies:

$$\{6, 0, 0, 0, 0, 0, 0, 3, 6, 6, 7, 0\} \text{ and } \{12, 0, 0, 0, 0, 0, 6, 6, 6, 0, 11, 3\}$$

We get the "old" example of a functionally closed set of Table 6.2, but also a new set due to the additional technology present in Table 6.3. Instead of using old technology, the new technology of the last column is used, and in order to do so,the technology in column 7 also has to be applied to generate the required inputs. Note that one functionally closed set does not exclude the use of the other: any linear combination of these two vectors will result in a combination of technologies that creates all of its own inputs, enough even to allow growth at rate α.

Table 6.4: Additions to the existing technology matrices of Table 6.3.

	new column	**new column**
products	*input matrix*	*output matrix*
2	1	0
3	0	0
5	1	1
7	0	0
11	0	0
13	2	0
70	2	0
110	1	0
154	0	0
67600	0	$\frac{31}{5}$
130	0	0
260	0	0
104	0	0

The invention of a new technology represented by the new columns in Table 6.4 provides a completely new way to assemble product 67600. The space of functionally closed sets is spanned by

$$\{6, 0, 0, 0, 0, 0, 0, 3, 6, 6, 7, 0, 0\}, \{12, 0, 0, 0, 0, 0, 6, 6, 6, 0, 11, 3, 0\}$$

$$\text{and } \{360, 240, 120, 120, 60, 0, 0, 0, 0, 0, 341, 93, 30\}$$

While the addition to the technology matrices in Table 6.3 only provides an alternative technology to produce product 67600, the new technology of Table 6.4

allows for a completely new combination of technologies to become self supporting.

With increased numbers of products and technologies, the need for an efficient algorithm very quickly becomes clear. The above examples are small, and create no problem at all. Even with the algorithm provided by Weil Jr. (1964) problems of this size can easily be addressed. However, when the product and technology sets expand a number of times and consist of a large set of products and technologies, the binomial coefficient $\binom{n}{m}$ becomes very large very quickly. The value of an efficient algorithm becomes clear instantly. Figure 6.7 shows the number of graph vertices known during the run of the algorithm that still need to be visited in order to assess whether or not they contribute a new basic vector.

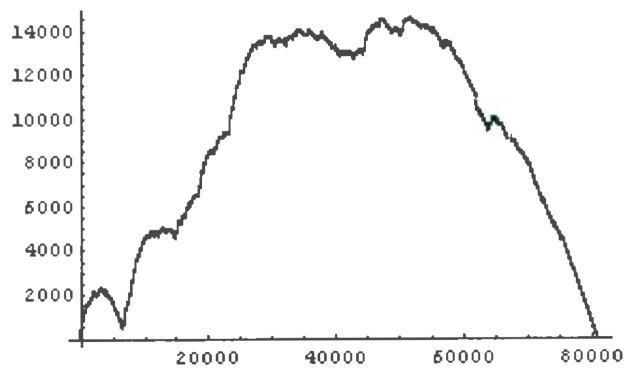


Figure 6.7: The number of known vertices that still need to be visited during the search of the graph.

With the number of products $n = 19$ and technologies $m = 29$ the number of combinations of n out of m equals $\binom{n}{m} \approx 20 \cdot 10^6$. The total number of vertices visited was around 80,000, which is approximately 0.4% of the total number of combinations. Nevertheless, it took an overnight run to determine the 100 plus basic vectors that span the solution space. Each of these vectors gives a set of activities that is closed, such that all required input is generated by the set itself.

and each cluster of activities is thus self sufficient. The resulting vectors are not included in this section, since they offer little extra information. There are many alternative production paths, and combinations of these, that only differ slightly in their composition. It will be more interesting when there are more distinct clusters to be detected. This can be achieved by ignoring particular types of goods. For example, a cluster of technology that is not completely self contained, but requires labour and raw materials from outside, can still be considered an organized unit of technology.

When functionally closed sets are defined as sets that are independent of outside input except for labour and raw materials, the presence of clusters can be analysed by means of the above method, except that now the return rate is no longer known. In this case, the complete Thompson algorithm has to be applied. This is where some practical issues arose. In the analysis above, the expansion factor is known to be equal to two, and thus the extreme points of the matrix game $M_2 = B - 2A$ could be determined to give the different combinations of technologies. Now we have an algorithm that first must determine the return factor, and only then can determine the extreme points. If the algorithm is applied with full precision to the rational representation of all the matrix coefficients, the algorithm becomes increasingly slow, the effect of horrendous fractions that make up the linear equation systems that need to be solved. When these fractions are simplified, we lack the required precision to determine the exact return rate, and this makes it impossible to precisely determine the extreme points that make up the different functionally closed sets. To this problem we have yet to find a solution.

Spatial clusters

The analysis could be extended spatially to see if there are functionally closed sets of agents that are spatially localised. For this to work, a better algorithm to determine the functionally closed sets has to be in place. In addition, it would be of more interest when an agent has the option of moving to a better suited location. As it stands now, a new agent chooses a random location or a location where most of his inputs are available. Once an agent is established somewhere, it does not move. This assumption can be relaxed.

6.4 Measures of evolutionary dynamics

Bedau *et al.* (1998) describes a measure to classify evolutionary dynamics. To that effect they developed a method to identify innovations that make a difference, where they "consider an innovation to "make a difference" if it persists and continues to be used" (Bedau *et al.*, 1998, p.229).

Evolutionary activity is measured by diversity, new activity and average aggregate activity. Based on these measures, Bedau *et al.* distinguish three classes of evolutionary dynamics: non-evolutionary, bounded, and unbounded (Table 6.5).

Table 6.5: Classification of evolutionary dynamics. Some models of artificial life show no evolutionary activity (class one), while some display bounded (class two) and others unbounded (class three) evolutionary activity (Bedau *et al.*, 1998) . In the headings D stands for Diversity, A_{new} stands for new activity, and A_{cum} stands for average aggregate activity

class	evolutionary activity	D	A_{new}	A_{cum}
1	none	bounded	zero	zero
2	bounded	bounded	positive	bounded
3	unbounded	unbounded	positive	bounded

The indicators D, A_{new}, and $\bar{A}_{cum}$ of Table 6.5 were determined for two data sets of fossil records, the Benton data set and the Sepkoski data set. These data sets record the taxonomic families. The Benton data set contains the fossil record of all families in all kingdoms, the Sepkoski data set contains the families of marine animals. This resulted in the graphs of Figure 6.8. The top graph shows the increase in diversity over time, where diversity is the number of taxonomic families. New activity (middle graph) indicates the appearance of new families, and mean activity (bottom graph) is cumulative activity divided by the number of families present at the time. Since diversity is increasing and mean activity is bounded, the total activity is unbounded, and thus the evolutionary dynamics as recorded in the fossil data sets belongs to class three. There exist few computer models capable of generating such dynamics (Bedau *et al.*, 2000)

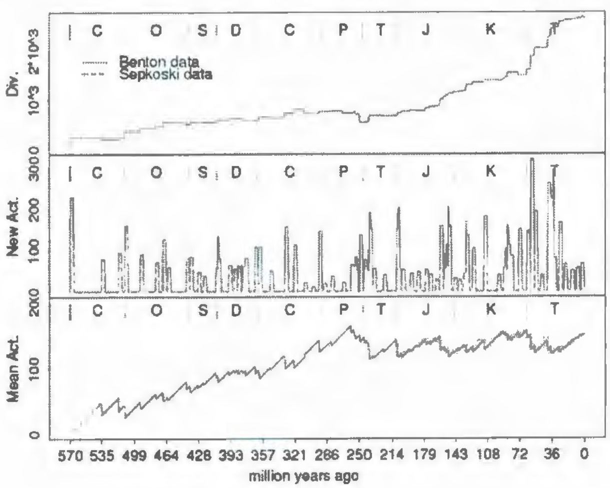


Figure 6.8: Bedau's analysis of the evolutionary dynamics of life as recorded in the fossil record data. Image copied from Bedau *et al.* (1998)

When one wants to apply this Bedau measure to assess the evolutionary qualities of a data set, two issues need to be resolved: what are the components that make up the innovations, and how should the evolutionary activity of a component be measured (Bedau *et al.*, 1997)? For the model in this thesis, an innovation consists of a new technology that appears as a new column of the von Neumann technology matrix. Products were also considered as components, but technological innovation does not necessarily involve new products, and thus as components products are less suitable than technology. The second question to be answered is how to measure the contribution of a technology to the system. As the quote at the beginning of this section states, an innovation contributes to the system if it persists and if it continues to be used. Bedau *et al.* have used a variety of measures of persistence, "depending on the nature of the components and the purposes at hand" (Bedau *et al.*, 1998, p.229). For the analysis of the fossil data sets they define the evolutionary activity of a component at time t by its age at time t, given that the component still exists at that time (Bedau *et al.*, 1998), and cumulative activity then is defined as the sum of evolutionary activity over all components. In "Towards a Comparison of Evolutionary Creativity in Biological and Cultural Evolution" Skusa and Bedau use a data set on patents, and the measure of patent activity they apply is the number of citations the patent has received in other patents up to time t (Skusa and Bedau, 2002). Indeed such a measure indicates the persistence of the component itself, as well as its importance to the subsequent components that make use of it.

For our technologies we can do something similar. A technology is persistent if it is still present, and if it supports subsequent (or any other) technology, meaning that there is demand for its products. A technology is active because the market is conducive, which is caused by other components. The higher its output, the more the technology contributes to the system in terms of creating opportunities for other (old and new) technologies. However, while a patent receives only one citation per subsequent patent and thus it is reasonable to cumulate the citations over time, a technology can be used over and over again. Therefore, we have defined the component activity of a technology at time t by the value of output generated by the technology during iteration t and not by the summation of activity from 0 to t. The aggregate activity at time t is the total output generated at time t by all technologies, the diversity at time t is the number of active technologies at time t and the mean activity is total output divided by the diversity.

In order to assess whether the type of evolving dynamics generated by the model in this thesis corresponds to real evolutionary dynamics, Bedau's measure was first applied to a simplified version of the model. The reason for simplifying the model is that the simulations over many iterations of an expanding economy required for such an analysis take a long time. For a faster version of the model, the agent based model of economic activity was replaced by an optimization routine. Instead of having each agent separately decide what the most beneficial actions (in terms of personal gain) are, the execution of a whole iteration was replaced by an optimization routine to calculate the required economic activity in order to generate sufficient resources to allow the maximum consumption of each of the agents. In other words, based on the occurrences of technology the economy, what is the maximum level of consumption possible?

In the case of Leontief-type economies, we showed above that aggregate agent behaviour can be predicted by the Leontief stable state, which is equal to the minimum economic activity required to generate the resources for the specified forced consumption. However, the modeled economy does not remain a Leontief-type economy: innovations in the form of additions to the von Neumann technology matrices occur at random times as before. Ideally, the more complicated economies that allow input substitution also maximize the use of technology. Ideally, the optimal economic activity level optimizes the use of the most efficient technologies. In reality, and in the model, this is not necessarily the case: the application of new technology, even though it is possibly more efficient than other techniques, depends on a number of factors, such as the availability of its inputs. By assuming here for the simplified version of the model that the global economy will indeed make use of the available technology in the most efficient way, we assume that individual agents in the more complicated model act in a similar way as the individual agent in the Leontief-type model behave.

The first results (Figure 6.9) indicate that the evolutionary dynamics of this model belong to class three, as diversity is increasing and mean aggregate activity is bounded, which leads to the conclusion that evolutionary activity in the simplified model is unbounded. However, this does not prove that the real agent based model shows the same evolutionary dynamics. This depends on whether the agent's behaviour per iteration is comparable to the calculated optimal behaviour in the simplified version.

In order to assess the assumption that the agents and price mechanism are capable of organizing the production system such that consumption is maximized with minimum activity levels, the production numbers per iteration were compared with the optimal production numbers as used in the simplified model. The resulting graphs show that, while it is not always the case, in many instances the dynamics of production numbers mirrored those of the optimal production numbers, although

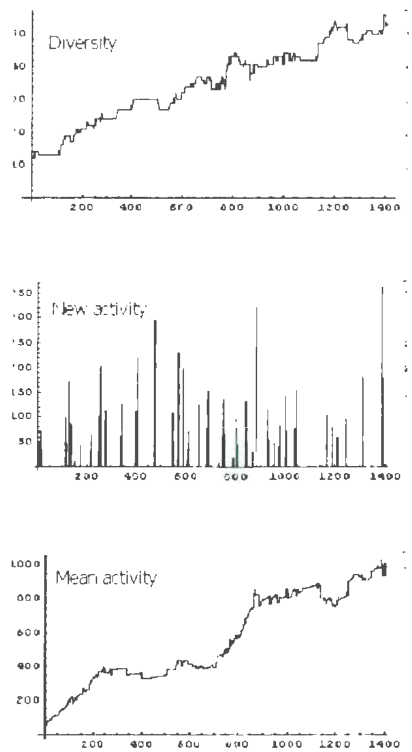


Figure 6.9: Bedau's measures applied to an expanding von Neumann technology set. The top graph displays the number of active technologies, the graph in the middle shows the activity of new technology, and the bottom graph displays the average activity per active technology.

often with a delay and on a smaller scale, see Figure 6.10. The delay is explained by the memory of the auctioneer in the price mechanism, who looks at the past to set the prices, while the optimal activity levels are independent of the history. The price mechanism aims to maintain sufficient stock for past production, and new technology has to appear in the history of the system in order to slowly make its way in, and this is why according to Schumpeter a visionary entrepreneur is required besides a manager (Schumpeter, 1961). There is no previous knowledge to base actions on. In addition to the predictable agent behaviour in the case of a Leontief dynamic system discussed before, this comparison provides some idea of how well the more complicated enlarged economy with input substitution and joint production behaves, and how capable the price mechanism is in allocating resources such as materials, time and agents. The fact that aggregate agent behaviour mirrors optimal behaviour to some extent justifies the applied simplification, enough to suggest that the evolutionary dynamics generated by the model fall into Bedau's class three. This in turn justified some long, overnight runs of the full agent based model to verify that the full model indeed generates unbounded evolutionary activity.

Figure 6.11 displays the three Bedau indicators for a model run in which there is an increasing number of agents due to the possibility of the introduction of new agents. The parameters of the Bruckner-inspired submodel dealing with technology field exchanges are defined such that a low occupancy of a technology combined with a high return rate results in a high probability of the introduction of a new agent who possesses this technology, similar to a region open to immigration. New technology does not have to be taken up by existing agents only, but can also be taken on by new agents. It is thus easier to add to the existing system, instead of replacing part it. An increasing number of agents makes it easier to expand the number of technologies used, and thus to increase the technological diversity of the production system. Indeed, Figure 6.11 shows an increasing diversity, while the added technologies result in a higher mean activity. That is, the output per unit of applied technology is increasing. This suggests that the application of additional technology has a positive effect on the production system. Thus, not just an increase in diversity because additional technology appears, but instead the increased diversity results in a higher output of the system. According to the Bedau measures, this is the

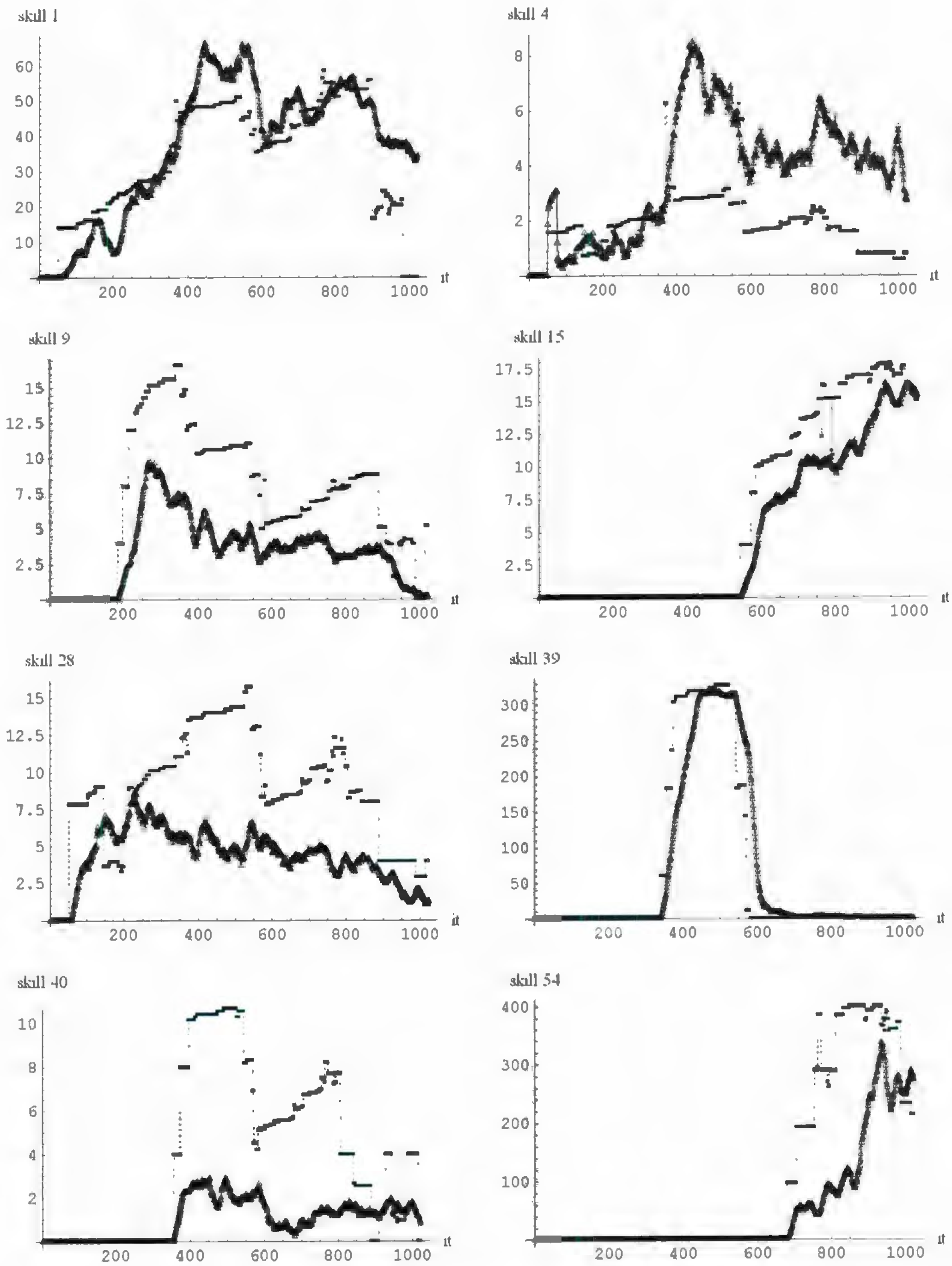


Figure 6.10: Production per iteration, averaged over 25 iterations to smooth the graphs, compared with the optimal production figures for the skill as calculated by an optimization of the labour output per iteration. Here 8 out of 66 skills are shown, of which some are good and some are bad fits. The squares display the optimal activity, and the triangles show the aggregated behaviour of the agents.

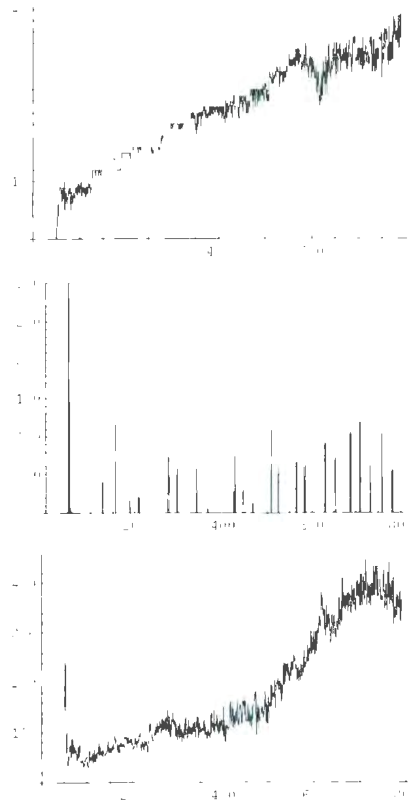


Figure 6.11: Bedau's measures applied to the economic activity of an agent based model with the possibility of immigrants supporting the successful skills. The top graph displays the number of active technologies, the graph in the middle shows the activity of new technology, and the bottom graph displays the average activity per active technology.

essence of evolutionary dynamics: the system generates new activities, and as the diversity increases, the new activities do not simply replace the existing ones, they have a positive effect on the system by increasing its output.

Figure 6.12 shows the results of a simulation in which only the existing agents are capable of imitating successful skills. The Bruckner-inspired submodel parameters are such that there are no immigrants that enter the system in order to strengthen the existing production system. Occasionally, there is a new agent with new technology, but this can happen only once per new technology, since after that the

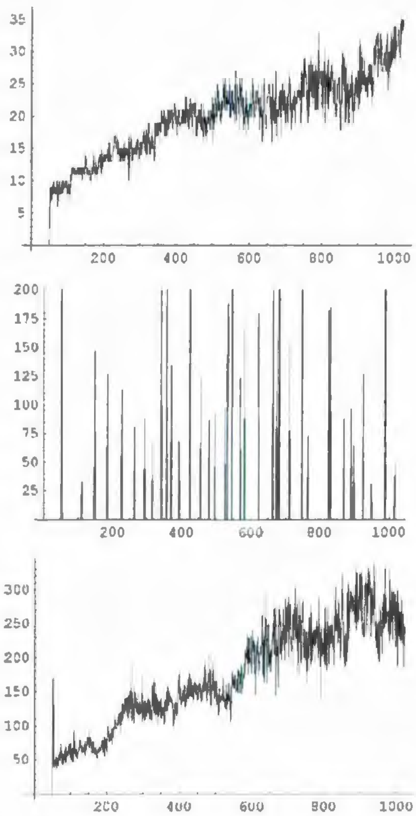


Figure 6.12: Bedau's measures applied to the economic activity of an agent based model without the possibility of immigrants supporting the successful skills. The top graph displays the number of active technologies, the graph in the middle shows the activity of new technology, and the bottom graph displays the average activity per active technology.

technology is no longer new. As a result, production capacity is limited, and if the system wants to adopt new technology, it will have to replace existing technology. The results indeed show that old, successful technology, once applied by numerous agents can fade, and is replaced by more advanced technology, with higher capacity, or better suited to the time, in that they require inputs and produce outputs that are more in line with the demand. Here, the first signs of creative destruction are clearly visible, although calling it a gale is still a little far fetched. This is only a matter of better programming and better hardware; the model provides an appropriate platform for simulating evolving economies. That indeed the artificial economies evolve is illustrated in Figure 6.12. At random intervals there are signs of new activity, this new activity leads to an increase in diversity, and again, the increasing diversity results in a higher output.

Chapter 7

A spatial application

So far space has received little attention. That is not to say that space has had nothing to do with the results generated up to this point. Agents have always been located on a grid, and their environment consists of a local environment. Social networks are not necessarily spatial. Nevertheless, such networks constrain the interactions to a limited part of the whole model space. In the case that an agent does not have access to all of the resources that serve as input for the technology it possesses, it is allowed to expand its network. The rules that govern this process can include spatial elements. For example, finding the nearest provider of a particular commodity, while removing the furthest trading partners, will lead to the most compact supplier group possible. Alternately, an agent can select one of its present contacts and investigate the direct surrounding of that contact in an attempt to add additional access to resources. Eventually this will result in a suitable group of trade partners, and per agent trade is thus limited to a specific part of the model. This leads to a limited local availability of goods, and in order for processes to function properly, goods have to be in the right location. Still, the role played by this spatial arrangement is limited. In order to illustrate that the individual based model is inherently spatial, this chapter will discuss two different implementations of transportation costs. For an alternative implementation of transportation costs in the von Neumann framework see Weil (1967).

7.1 Transportation costs

Transportation costs imply that the price of a commodity acquired at a different locale than where one is situated increases per unit of goods and per unit of distance. In comparison to the model of the previous chapters, this means that when an agent surveys its network of trade partners, it not only has to know how much is in stock, but also where, since the distance to each location comes with an additional cost. So far, an agent acquired a specific commodity from one agent at a time. Each turn, an agent had one supplier for each of the acquired goods. This implies that the constraints that applied to the purchase of a specific good were given by the partnering agent possessing the largest quantity for sale. How much other agents possessed, and where the agents were located was irrelevant. As mentioned earlier, this implementation of the buy action was mostly driven by the simplicity of the construct, making it easy to implement and more importantly, fast to execute. However, there are more realistic, and possibly also more interesting, implementations. This section will describe the changes required to include transportation costs in the simulations of an agent based market model.

Remember that the total range of actions that an agent can perform is given by

$$
\begin{aligned}
i &= (\; A \mid M \mid I \;) \\
o &= (\; B \mid I \mid M \;)
\end{aligned}
$$

where the production input matrix and output matrix are given by A and B. I

and M are also input and output matrices, not for producing, but for trade. The input for buying a product is its price, which is expressed in a number of units of a product designated to serve as money. The output of that action is one unit of the desired product. Selling simply reverses the input and the output vector of buying: the input for a selling action is a product and the output is its price. Dimensions of matrices A and B are $k \times l$, meaning that there are k production processes and that the economy consists of l products, with $k, l \in \mathbb{N}$. The first product is labour and the second product represents one unit of money. These matrices are joined by the matrices M and I: I is the identity matrix of size l. M is the square matrix of size l with all elements equal to 0 except for the second row, which is equal to $p = \{p_1, p_2, \ldots p_l\}$. Thus, i and o are the input and output matrices for manufacturing, buying and selling combined. The use of this extended input and output matrix is explained in Section 3.2.3.

Selling plays a secondary role in the model. It never results in a profit, since an agent values a product at the current price and has no memory of the price for which the good was acquired or manufactured. Selling is used to generate revenue to acquire the inputs needed to make profit, and a trading partner with enough monetary resources will buy the goods for sale. As such, transportation costs applied to a sale are left out of consideration here, and the input and output of selling I_s and M_s remain the same as I and M above.

$$
\begin{aligned}
i &= (\; A \mid M_b \mid I_s \;) \\
o &= (\; B \mid I_b \mid M_s \;)
\end{aligned}
$$

To implement transportation costs in buying transactions, the matrix M_b, representing the monetary input of all purchases must have a column, not just per product, but per product per trading partner. Trading partners in different locations will have different transportation costs, and this is represented by the enlarged buying input matrix M_b.

There are different ways to deal with transportation costs and the next two sections show two different implementations.

7.1.1 Transportation cost as increase in price

Let the product set be $\{2, 3, 5, 7, 11, 13, 70, 110, 154, 130, 260, 104\}$, where 2 plays the role of labour and 3 equals a unit of money.

Table 7.1: The possible transformations, where the output coefficients are determined such that all sectors have an equal profit rate of 50%.

products	*input matrix*										*output matrix*									
2	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	2
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0
11	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
13	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	$\frac{1}{5}$	0	0	0
110	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	$\frac{4}{5}$	0	0
154	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	$\frac{1}{5}$	0
130	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	3	0	0	0
260	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	4	0	0
104	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	$\frac{531}{256}$	0

With this set of manufacturing rules comes a price set $\{1, 1, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, 0, 6, 6, 6, \frac{5}{4}, \frac{27}{32}, \frac{1}{3}\}$. Before, the matrices M and I, containing the input and output for purchases, consisted of a separate column for each of the products. For example, the input for the purchase of one unit of product 5 is $\{0, p[5], 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$ and the output is $\{0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$. However, when the model differentiates between suppliers to enable transportation costs to play a role, and these transportation costs are included in the price.

$$\{0, p[5] + r \cdot d_i^n, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

becomes the new input vector for a purchase of one unit of product 5 from trading partner i, at distance d_i and distance exponent n, with r the transportation rate, i.e. the cost per unit product per distance. Each trading partner obtains a separate input vector for each of the products. Transportation costs have to be incorporated into the model even before an active agent plans its action for its turn, because the agent needs to have the resources to cover the transportation costs. When an agent maximizes its actions such that it makes the most profit possible, it will use all available resources in the most optimal combination. Therefore, a late billing of transportation cost will put an agent in debt, which violates the assumption of always maintaining a positive balance. The inclusion of transportation costs in the input and output matrices, such that the agent takes these extra costs into account before deciding on the best combination of actions, prevents such a debt to occur.

Besides adding an important spatial quality to the model, this approach allows for more realistic treatment of the selection of providers. Even without transportation costs (r is equal to 0), this approach allows the agent to buy from any supplier available, since the agent is not limited by the one supplier per turn constraint applied before. However, this additional feature comes with a price in terms of modelling efficiency. The parts M_b and I_b of the matrices i and o are much larger, resulting in many more options for an agent when it plans for maximal profitable action. The constraint that one commodity can be purchased from only one agent per turn certainly was a very efficient way of implementing the trade. The model can be fitted with more advanced rules governing trade, as was just explained. The next section explains a different interpretation of the transportation costs.

7.1.2 A transportation sector

Instead of directly increasing the price of a product when it is purchased over a further distance, it is also possible to create a transportation sector. One good is designated as being involved in transportation, and for every unit of product purchased, one needs to arrange $r \cdot d_i^n$ units of the transportation product. For example, $\{0, p[5], 0, 0, 0, 0, 0, 0, 0, r \cdot d_i^n, 0, 0, 0\}$ is the input required for the purchase of one unit of product 5, of which action the output vector is $\{0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$. Instead of paying the supplier an increased price, and thus assuming that the supplier will take the time to deliver the product to the buyer's location, the buyer now has to arrange the availability of a number of units of an additional good (e.g. a rental truck). In Table 7.2 a new product 10010 has been introduced and with each purchase from a location other than where the buyer is located, a number of product 10010 has to be arranged (i.e. produced or purchased). An agent specializing in providing 10010 can be considered an agent in the transportation business.

As can be seen in Table 7.2, product 10010 is generated by technology seven (the seventh columns of the input and output matrices), and product 10010 is not used for anything in the manufacturing process. The role of this product is currently limited to transportation. However, it is not necessary to introduce a new product to play the role of transportation mean. Below one of the existing products is designated as a transporter.

Table 7.2: The addition of a new product that plays the role of a means of transportation. The product has no other use, although that can easily change in future new combinations. It consists of the products 5, 7, 11, and 13 combined with labour 2 into product $2 \cdot 5 \cdot 7 \cdot 11 \cdot 13 = 10010$.

products	*input matrix*											*output matrix*										
2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	2
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
70	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	$\frac{1}{5}$	0	0	0
110	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	$\frac{1}{5}$	0	0
154	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	$\frac{1}{5}$	0
10010	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
130	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	3	0	0	0
260	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	4	0	0
104	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	$\frac{531}{256}$	0

7.2 Results

7.2.1 Initial experiments

The experiments based on the transportation model outlined in the previous section involve two changes to the model. First, in order for an agent to select a supplier when there are possibly multiple suppliers at different locations per turn, an agent has to know who and where all suppliers are. The choice of supplier is not so much made based on who is in its social network, but based on which supplier is nearest such that the purchase involves the least transportation costs. Here, the spatial structure upon which the agents base their decisions is purely defined by distance, and no longer by the construction of a network of trade partners. Of course one could combine these two spatial arrangements, but for simplicity here we have chosen to let the selection of trade partners be fully dependent on the distance. This implies that every agent has to know about every other agent that is possibly relevant. The algorithm that defines the agents' strategy to expand the network thus has to be different from the algorithms outlined in Section 3.2, as trade over distance is not constrained by the compact network, but by transportation cost considerations.

In this chapter it is desirable to have every agent connect to every other agent that can be of any relevance to it. Every supplier producing something that serves as input for an agent's technologies is relevant. Furthermore, to assess the local market in terms of supply and demand to set the local price, an agent needs to be connected to its competitors in order to decide if the market is saturated with its own output or if there is demand for more. Based on these considerations the following algorithm for network expansion is applied:

1. Determine all products that serve as input
2. Select the nearest provider of a product that is not yet secured
3. Once providers for all inputs have been secured:
   (a) Expand the list of relevant products with output(s)
   (b) For one of the products from the list in 3(a) select the nearest producer not yet in the network and add this agent to the network

Every time an agent's turn comes up during the simulation it will execute the above algorithm and over time it will obtain a connection with every relevant agent, and it will keep up with the addition of new agents if this is the case.

The second important implication of the transportation model applies only to the transportation sector model where a specific product is responsible for the additional cost of transportation. Where before there was only one type of external demand, being the exogenously determined fixed rate of consumption of consumers to maintain their good health, the inclusion of an additional cost for transportation presents a second "drain" on the system. (In the other transportation cost model variant, where the transportation cost is included in the price, the provider is assumed to deliver the product at the buyer's door, for which the provider is rewarded by receiving an elevated price. The extra costs however do stay in the system.) In the case a particular product is designated as being used up during transportation, a certain quantity of this product is removed from the system with every transaction between different locations. The quantity of product that leaves the system depends on the spatial configuration, as trade over longer distances involves more transportation, but even more importantly, the quantity of product that exits the system depends on the economic activity. The more trade there is, the more dissipative the system becomes, and thus the more activity is required to keep up with demand. This positive feedback requires careful attention to the stability of the system.

In order to prevent large deficits, which can and will crash the system, we need to pay extra attention to the history of a simulation at the start-up. As explained earlier, the history of w iterations is used to determine an appropriate level of action and thus an appropriate level of stock. While before the history of a model at start-up was empty, the model was nevertheless capable of generating a rather stable production process with appropriate levels of activity, where the appropriateness is assessed by the similarity to the calculated Leontief activity levels. However, as Figure 3.3 illustrated, in such a set-up there is a certain amount of surplus production following the too low production caused by the initial empty history, resulting in a cyclical pattern of over and under production. To prevent this initial imbalance, instead of an empty history, the activities during the $w = 50$ iterations before the start of the model are given by the activity levels required for a certain external demand as calculated by the inverse Leontief matrix. This indeed leads to a more stable system, see Figure 7.1.

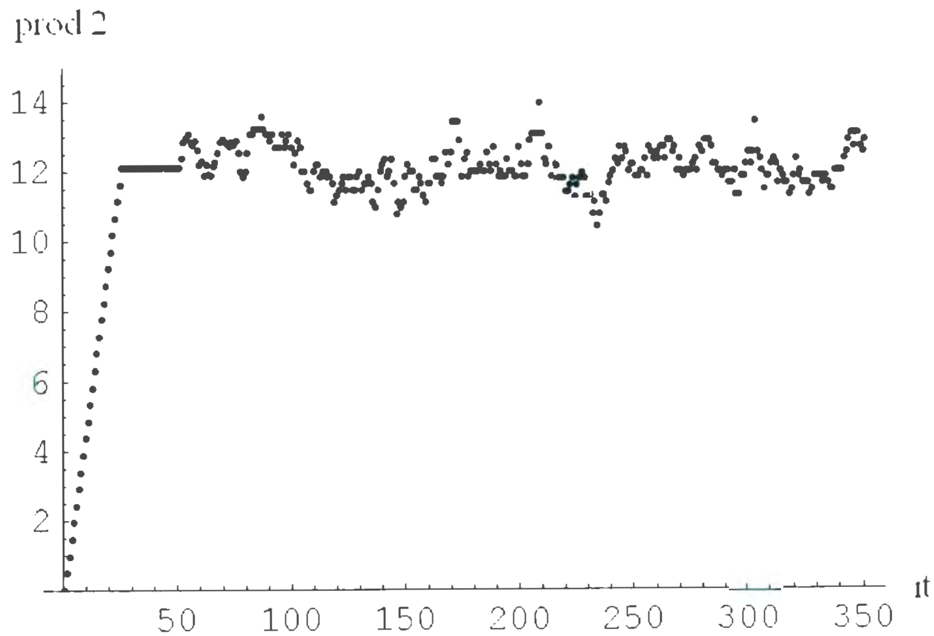


Figure 7.1: The average production of product 2 (labour) per iteration, averaged over a window of length 25. When the history at the start up of the model is given by the calculated Leontief activity levels, a more stable production is established more quickly than for a simulation starting with an empty history. For the given external demand the calculated production level for product 2 was slightly higher than 12.

With the introduction of a transportation sector a new challenge arrives. As mentioned above, the expenditures on transportation form a new source of external demand, a demand that does not originate from production. What makes matters worse is that this external demand is not fixed: the total required quantity of transportation depends on the spatial configuration and on the activity levels. For example, the small system above with exogenously set consumption level $\frac{1}{5}$ and 8 consumers has a Leontief output of

$$\{\tfrac{8}{5}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \cdot L^{-1} \cdot output\ matrix \approx$$

$$\{12.1, 0.0, 1.3, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 2.6, 5.3, 10.6\}$$

while an extra cost of 1 unit of product 11 spend on transportation would require

$$\{\tfrac{8}{5}, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0\} \cdot L^{-1} \cdot output\ matrix \approx$$

$$\{19.7, 0.0, 2.1, 0.0, 1.0, 0.0, 0.0, 0.0, 0.0, 4.3, 8.5, 17.0\}$$

and when a more expensive product is designated as required for transportation, such as product 154, even if the total use per iteration, thus over all agents is only 1 unit

$$\{\tfrac{8}{5}, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0\} \cdot L^{-1} \cdot output\ matrix \approx$$

$$\{34.8, 0.0, 3.8, 1.0, 1.0, 0.0, 0.0, 0.0, 1.0, 7.6, 15.1, 30.2\}$$

The latter example shows a threefold increase in required labour over the first example. It illustrates very well the additional activity required when there is some form of transportation cost involved, and the question arises whether or not the price mechanism is able to guide the model through such a wide range of levels of activity. During a simulation the model will not change from one without transportation costs (as in the first example here) to one with product 154 as required for transportation (the latter example). However, one moment there is sufficient stock of product 154 and it does not have to be produced in order for transportation to be possible, the next moment this extra unit of product 154 needs to be coughed up by the system, and the activity levels are required to be much higher.

On top of that, the increase in activity most likely requires additional means of transportation, requiring even more activity. The addition of a transportation sector to represent transportation costs in the model, in appearance rather simple, becomes a very demanding part of the model. That is not to say that it does not work, some experiments including a transportation sector led to a sustainable economy. However, the same simulation with merely a different initial spatial configuration of agents can easily be one that crashes because the agents cannot keep up with the demand. Once a system runs out of transportation means, trade is limited to trade between agents in the same location. This usually implies a crash. The same happens when production levels fall too far behind. The chance of such crashes can be reduced by creating a larger buffer, either through increasing the stock levels that the auctioneer perceives as a normal stock and thus moving away from a "just in time" economy, or by increasing the number of agents through which there is a larger number of bodies that can intervene (which at the same time implies the presence of a larger stock). The examples of simulations that follow have a rather small contingent of agents due to the lack of computational power at the time of running the experiments.

We continue the simulation with the calculated history of Figure 7.1 above, with the difference that this time transportation costs are taken into account. The transportation costs per unit are the same for each product, and depend on the distance. Here we have chosen to have the costs not to go up linearly with distance but with an exponent greater than 1

$$transportation\ costs = q * r * d^{3/2}$$

where q is the quantity purchased, r is the cost per unit per distance and d is the distance between the two trading agents. The transportation cost is then implemented as explained in the previous section, thus in the purchase of a good the total costs are $\{0, q * price, 0, 0, transportation\ cost, 0, 0, 0, 0, 0, 0, 0\}$. Here we have chosen the fifth product, product 11, to be the one representing the means of transportation.

The simulation is initiated with the same agent and technology distribution as above (Figure 7.2). The layout of the producers is contrived in such a way that there are a number of producers clustered together in the middle of the map, and a small number of consumers are scattered around, some near the center, others further away. The external demand now is not just made up by the obliged consumption of agents, but also by spending product 11 on transportation. The fact that it is unknown how much transportation is going to be required complicates matters further. However, the quantity of product 11 required can be estimated by running a simulation to obtain a rough indication. On average around 2 units of product 11 are spent on transportation per iteration. This figure gets us in the ballpark and the corresponding calculated history is

$$\{\tfrac{8}{5}, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0\} \cdot L^{-1} \cdot output\ matrix \approx$$

$$\{27.3, 3.0, 0.0, 0.0, 2.0, 0.0, 0.0, 0.0, 0.0, 5.9, 11.8, 23.7\}$$

When these activity levels are used to populate the history of the model, an eventually stable production appears, see Figure 7.3. The other production processes display comparable behaviour.

However, it is important to state here that the introduction of the transportation cost into the model does make the model far more unstable than before. The same simulation with merely a different initial distribution of agents often is unstable. Obviously the different spatial configuration makes for a different need for transportation, and thus the calculated history might have to be replaced with a

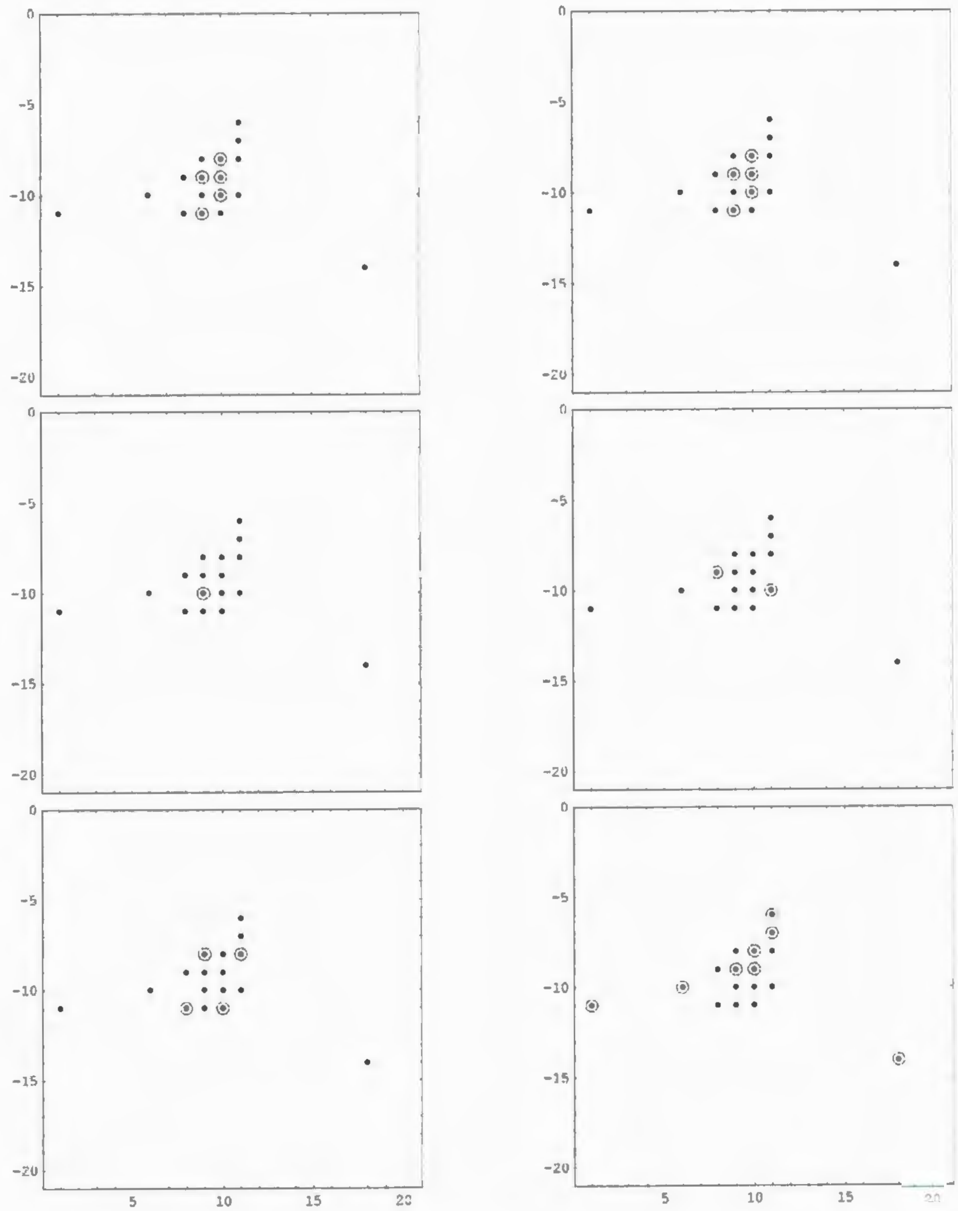


Figure 7.2: These figures illustrate the locations of the different producers. Every map shows all agents and indicates an agent with a specific technology by a circle surrounding the agent. From left to right, top to bottom, the figures show the producers (circled) of product 5, product 11, product 130, product 260, product 104, and finally product 2.

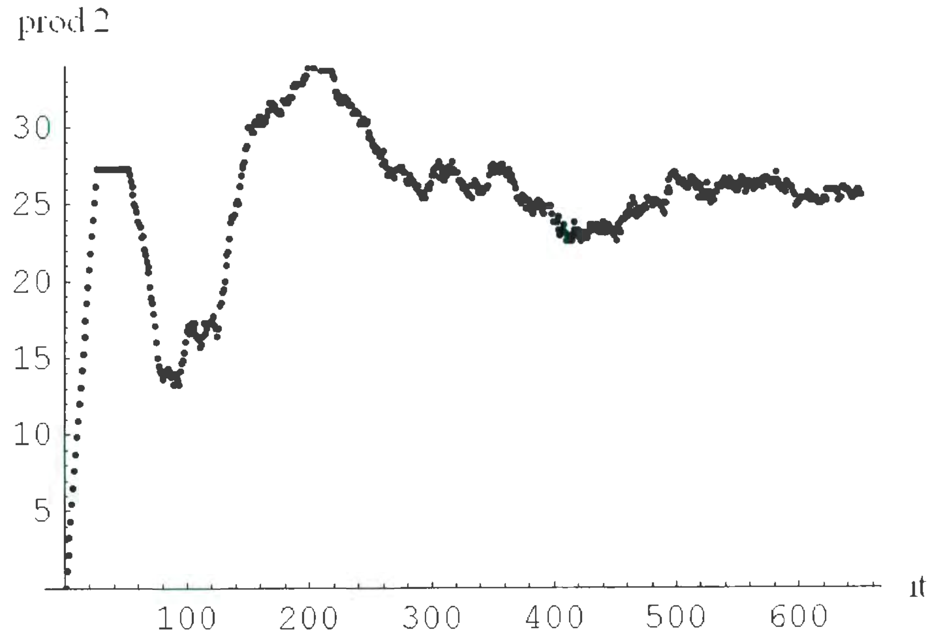


Figure 7.3: Production of labour in a transportation cost model. Apart from a big swing appearing just after the start of the simulation, the production of labour reasonably stable and corresponds to the predicted level used for "populating the history".

more appropriate one, but these experiments were not always successful and many simulations end in a crash. This instability might be partly due to the small class of agents that is modelled here, but at the time of the experimentations there was no more computer power available. Having said this, the simulations that are stable display interesting behaviour, some of it predictable, some not so much.

For example, when we look at the accumulation of transportation costs per location, the simulation shows results that fit our expectation. Figure 7.4 illustrates that most transportation costs are accumulated by agents further away from the center, which of course is hardly surprising.

However, Figure 7.5 shows the accumulation of money by each of the consumers.

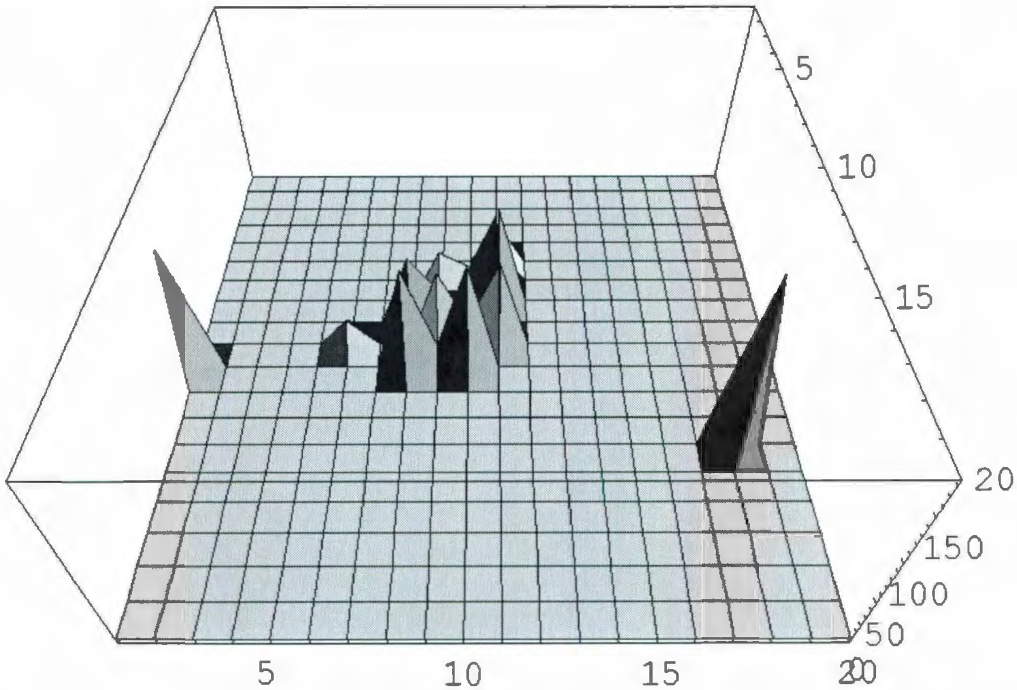


Figure 7.4: Accumulation of transportation costs by the agents. As can be expected the agents furthest away from the center collect the largest transportation expenses.

Surprisingly, the consumer the furthest away from its suppliers has obtained the most money, where at the beginning of the model all agents start with an equal amount. Such results are far from realistic, but they can be explained as follows: Under normal circumstances when the prices are near or at the equilibrium price levels because supplies are locally sufficient none of producers has to go very far to acquire labour. In these circumstances consumers further away have the opportunity to stash away a little stash of labour for when times are tough. When such times arrive and prices increase, the local suppliers run out quicker, since they never had the opportunity to increase their stocks. As a result the producers will have to go further for the required labour, at higher transportation costs. The ultimate effect of the transportation costs is that consumers (and producers alike) located further out only trade when nearer suppliers are out and effectively only when the price is high. In itself this is not entirely unrealistic, if transportation costs would counterbalance the higher prices. However, here this model shows a strong imbalance, the transportation costs are small in comparison to product prices. At first sight the solution to this imbalance is to increase the transportation costs by designating a higher priced good as the one that represents transportation. However, this is easier said than done, because as was illustrated earlier, when additional surplus that will exit the system has to be generated, the required activity levels go up rather fast and quickly stretch the production system.

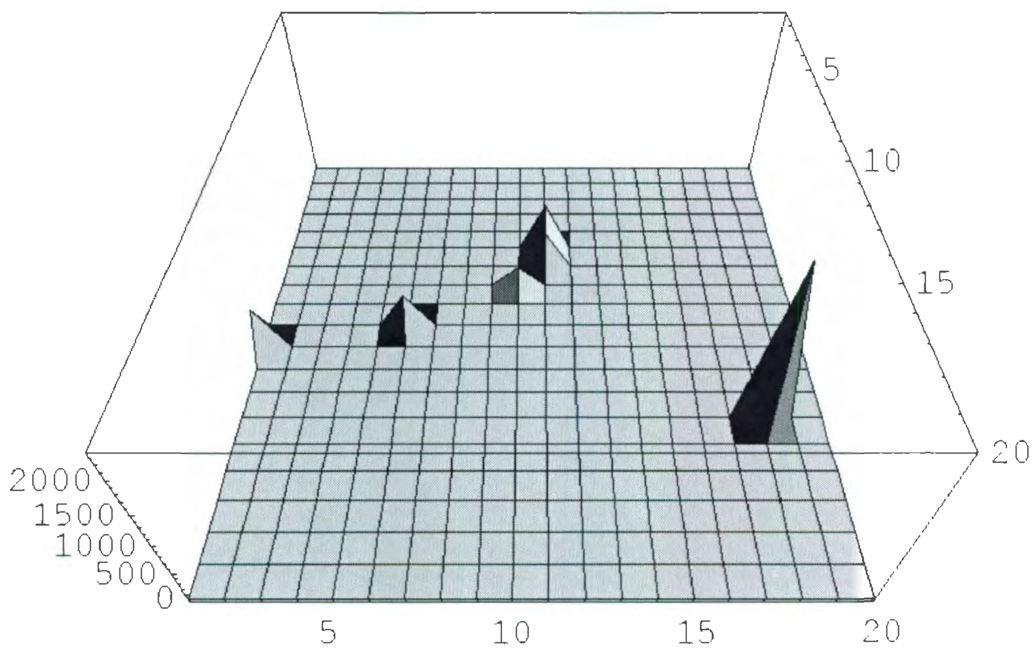


Figure 7.5: Accumulation of money (product 3). Surprisingly the agents further away fare surprisingly well, even though they do have the largest transportation expenses.

The problem sketched here is even more present in the other transportation model, where transportation cost is included in the price as if the producer is delivering and is compensated for the trip. In addition to the higher prices producers and consumers further away get due to the more pressing market situation, they also get a higher compensation for the transportation to the purchasing agent. In this model the imbalance is even bigger.

Chapter 8

Discussion and conclusion

The subject of evolutionary economics is very large in scope, and there are many perspectives on how to approach this subject. Schumpeter approached it descriptively, while in the nineteen fifties, sixties and seventies researchers (e.g. von Neumann (1946); Kemeny *et al.* (1956); Hamburger *et al.* (1967); Gale (1968); Morgenstern and Thompson (1976)) tended to approach evolution quantitatively, in terms of expansion, rather than qualitatively, in terms of increasing complexity. Evolutionary economics in the eighties and nineties broke away from the equilibrium paradigm that dominates economic theory, dropping many of its unrealistic but convenient assumptions, and instead focused on dynamic modelling of the process based on changing technological scenarios (Nelson and Winter, 1982; Jovanovic, 1982; Dosi *et al.*, 1995, 1997). Technological change in these models is pre-determined, or it is left undetermined and exists merely in terms of gains in productivity. Such a system, with quantitatively changing model coefficients, can simulate the resulting changes in economic interactions.

This thesis adds to these perspectives in that it acknowledges the central role played by technological developments, while at the same time accepting that such developments are more than mere quantitative changes, but can not be determined *a priori*. We see novelty in new products, and novelty in new production methods. However, it is unknown what the future will bring, and not just in terms of normal uncertainty. Not only do we not know which of the outcomes of a probabilistic process is going to occur, but we do not even know the set of possible outcomes (Knightian uncertainty (Kauffman, 2000, p.215)). Nevertheless, we can obtain an understanding of the interacting processes of the such systems through simulation. Just as artificial life attempts to simulate the evolutionary process of biological systems in order to gain insight into the relevant machinery without actually reconstructing any part of the real biosphere, so "artificial economics" can provide insight into the processes shaping economic evolution without predicting the details of that evolution. It is clear from the outset that economic evolution exists. Even in my short life span of thirty three years very big changes have occurred in terms of new products and new processes. Changes in communication technology, information technology, and energy supply have led to very different economic flows and dynamics than those of just 30 years ago. Yet economics does not know how to explain any of this. The equilibrium paradigm tells us what system state we can eventually expect given that everything else remains equal, but often times, before the stable state appears, the assumption of unaltered factors and circumstances has already been refuted because the circumstances have changed. So how can we better understand economic systems that do allow changing conditions?

In addition, not only is novelty important in that it proves standard assumptions wrong, it also is a driving force in economies (Dosi *et al.*, 1997; Dawid, 2006). New consumer products such as High Definition television or iPhones, plus a considerable effort by the marketing division (in itself another example of an innovation) to convince the consumer group of the need for such products, are indeed a driving force in our western economies, where much spending is in the form of discretionary consumption. How can we better understand economic systems that are endogenously driven by the introduction of new elements?

8.1 Discussion

The answer to the two questions posed above is, as this thesis has shown, by modelling novelty, and subsequently incorporating this novelty into an explicit market model in order to establish a realistic fitness measure. This affects the copying process of technology and, indirectly, the manufacturing of products.

In implementing this framework, many choices had to be made, and in making these choices, existing research was used as much as possible. Where this was impossible, hopefully a modeller's common sense prevailed. Many of the implementations required a little experimentation and fine tuning of parameters, but ultimately, the results were quite satisfactory, as Chapter 6 has shown. Regardless, there are many more possible implementations of this framework, and these still need to be explored. The present section will discuss some of the most pressing issues.

Innovation leading to price reduction

When invention occurs, new technology, and often also new products, are added to the existing set of technology and products. The addition is in the form of new columns and rows of the von Neumann technology matrices. The inputs for a particular transformation are fit, in that the inputs together will contain sufficient integer factors in order to compose the output. Even though this somewhat limits the possible combinations of products that can be transformed into more advanced products, the quantitative aspects of this new transformation are still undetermined. How much of each of the inputs is required and how much output is generated is unknown. When left to a random process, this could very easily lead to very unrealistic input and output numbers, e.g. one process being almost infinitely more capable than others.

Therefore, to keep the input and output coefficients in check, the rate of return on input value was kept constant for all processes. However, this does not imply that all technology is equally fit in every situation. Higher value of inputs for some processes combined with a fixed return rate necessarily leads to higher output coefficients. What this means is that even though the return rate is fixed for all technology, one technology can still have a higher output coefficient per unit of activity, and in terms of generated output, it will perform better than the technology with lower input cost and lower output. Since activity is bounded per time step, high efficiency technology is preferred, so that even though return rates are equal, there is a difference in performance. This often leads to the system replacing old technology with new technology. The success of higher performance technology depends on the availability of its higher valued input. Higher value input implies that more components are involved in the production of that input, and therefore that the assembly of these components may be more difficult to arrange. Thus, higher performance production comes at a price, and as such, is not necessarily successful. In this respect, having constant von Neumann return rates is not problematic.

However, the single fixed constant return rate for all processes has another consequence. In reality many innovations consist of lowering input coefficients or raising output coefficients, such that the new process leads to a lower price for the product.

This kind of innovation is currently excluded from the model because either of the actions would introduce a technology with a return rate different from the fixed model return rate. In addition to the fact that innovation in reality often results in improving return rates, having different return rates for different sectors is more realistic than the fixed von Neumann return rate. There is reason to remove this model assumption, but so far this has not been done.

Consumer behaviour

Two different types of consumer behaviour have been described in this thesis. One method prescribes a fixed rate of consumption for each of the consumers, regardless of the actual activity in the labour market. This type of discretionary consumption is equivalent to the metabolism of a biological organism. The problem with this approach is that with a fixed rate of consumption, there is no need for more advanced products. Consumers satisfy the demand for consumables, and when new consumables appear on the market these are ignored, since the fixed exogenously determined consumption was already met by the previous consumables. Many experiments have taken place with an increasing rate of discretionary consumption as more advanced production techniques and consumables appear on the market. However, it turned out to be difficult to find the correct rate of increase; with too fast of an increase the system grinds to a halt, since too many resources are wasted on consumption instead of manufacturing. If the increase is too slow, the interest in new technology and new products fades. This phenomenon may, however, be realistic.

An alternative consumption mechanism was implemented to encourage the production of more advanced consumables. With this mechanism, all consumers maximize the total labour generated according to their individual abilities and other constraints, as described in Section 3.2.3, regardless of an excess supply of labour. High demand for consumables in turn leads to high production numbers and thus high labour demand. Demand is guaranteed, and so there is always a need for more advanced production technologies, which will drive technological development.

Because all consumers maximize the quantity of labour they generate, and because more advanced consumables generate more labour, consumers prefer advanced products over more primitive products. This is not unrealistic, but at the same time, this is where there is room for improvement. Some form of diminishing marginal utility for consumables would be a more accurate implementation of consumer behaviour, for example. Instead of more advanced products completely taking over from the older products (to the extent that supply is able to keep up with demand), this would lead to a more realistic dynamics of demand, where the total consumption is spread out over several consumables.

Multi sector models

The number economy allows for separate sectors that do not overlap. By choosing different groups of primes that lie at the basis of each of the sectors, products of one sector cannot contribute in the manufacturing of products from another sector, except that they can perform as capital. Such a setup allows the modeller to take on a complete economic model, where developments in one sector through minimal but essential links can impact other sectors. The connectivity of our economic system is amazingly complex, and small changes can cascade into large phenomena. Jain and Krishna define keystone species as elements whose removal changes the organisational structure drastically (Jain and Krishna, 2002). The model could help identify such key elements in the system.

Reproduction of technology per field or per agent

Section 4.3 described how the occurrence of technology changes probabilistically. The probabilities governing these changes are determined based on global measures of success, connectivity and distance. Subsequently, a random draw selects a change to the technology fields. For example, technology i obtains an additional representative, to the cost of technology j. To implement this proposed change, the model finds the agents possessing technology j, but not possessing technology i. One of these agents is chosen randomly, and the proposed replacement of technology j by technology i is applied. This method completely disregards the possible negative or positive effects this change can have on the functioning of the selected agent. Possibly the agent has no use for technology i, or perhaps its main source of income was based on the use of technology j that now has been taken away.

An alternative to this rather crude implementation of reproduction of technology, which has not been implemented in the context of this thesis, is to couple the generation of additional agents or technologies to the agents themselves by giving each agent some set of rules that would allow the agent to decide which technology it would like to acquire, based on local circumstances. If an agent is profitable, what would be the best way to invest the gained assets? This would be more in line with the approach advocated by Nelson and Winter (1982), in which firms decide whether or not to expand or contract their assets or to change their routines based on the market situation they find themselves in. This certainly would improve the model, but, since it was not strictly necessary for the framework that is discussed in this thesis, the simpler mechanism was used (Bruckner *et al.*, 1989, 1996).

Intelligence of agents strategies

A similar remark can be made about the price and market mechanism. Currently, this is rather simple, and agents are merely price takers. However, as the resulting stable Leontief behaviour indicates, the price mechanism is effective in signaling shortages and surpluses that need to be addressed, and suffices to support the model. There already exist more advanced pricing mechanisms in the context of agent based modelling. The more advanced agents in those models set their own prices, which result from rounds of negotiations. Agents can have various degrees of intelligence and may be capable of planning for future use (LeBaron, 2001).

Spatial behaviour

In order to develop the model framework, and in order to keep the first model manageable on a normal desktop computer employed by a autodidact programmer, space was not given the full attention it deserved. However, the framework developed is flexible, and in order to investigate spatial aspects of the model proposed here, the model can easily be expanded. This really would make it a model for evolutionary economic geography. Currently all agents have a network of agents with whom they engage in trade. This network can be defined or constrained in any conceivable way. It is possible to have only very local interactions in a Moore neighbourhood, consisting of the central location and the eight surrounding grid locations. Or, very differently, the agents can have a social network that connects them to every other agent they know, regardless of the distance between them. This is the network scheme mostly applied for the examples in this thesis. Chapter 7 described a first experiment that includes transportation costs but there are many more experiments worth looking into. When agents are made mobile, what rules would best describe their behaviour? How would rising transportation costs affect their behaviour?

Run time

The simulations are generated on normal desktop computers, with a modest amount of RAM and a modest CPU speed. One of the processors used has a clock speed of 1 GHz and the other computer has a dual 2.1GHz processor, of which only one is used to do the calculations. Given the limitations of these machines, the size of the simulations (i.e. the size of the map, the number of agents on it and the number of technologies) were controlled by the patience of the modeller. The faster computer was used to run slightly bigger models, making the time per iteration basically the same for both computers. The patience of the modeller was tested thoroughly, and the results discussed in this thesis go as far as both the computers and the modeller could be stretched. More simulations and analyses are desirable but somehow first the run time required to do so has to be reduced.

8.2 Conclusion

Though there are still a number of issues that need improving or further exploration, the framework developed in this thesis is one with many merits. The main virtue is that this framework demonstrates that it is possible to treat the economic system as a constructive dynamic system. The field of evolutionary economics as it has developed over the last two decades already proved that economics can be treated dynamically, and that the adaptive qualities of elements are important in explaining typical patterns. This thesis adds the constructive character of the economy to the modelling framework, something that is equally or even more important to the evolution of an economic system. Despite the fact that the constructive dynamic model research applied in economics is only just beginning, and that some aspects need to be explored in more detail, it is argued here that the model has justified itself. The framework developed here is capable of dealing with endogenous change, something that is very important in our economy, but that until now has been essentially ignored in research. The application of a constructive dynamic system to the field of economics opens up new opportunities in economics, and once a proper theory of economics is in place this can be extended to economic geography.

In addition, the application of the Bedau measure indicates that the evolutionary economic dynamics generated by the model is appropriately named evolutionary: the dynamics display characteristics comparable to the case that defines our concept of evolution, the evolution of life in our biosphere.

Furthermore, the model shows that it is possible to simulate a free market economy, in which agents with very simple rules to determine their behaviour are capable of generating a well functioning economy. Resources and time are scarce goods, and thus need to be used appropriately for the system to function. Agents who perform in local environments do not know how the global economy is performing. Nevertheless, through a local price mechanism signaling where local surpluses and shortages exist, agents are capable of locally balancing the supply and demand. Since the set of local markets completely covers the global market, this leads to stable, well balanced global behaviour. Here we see Adam Smith's invisible hand at work in a dynamic spatial model.

The framework is also very useful to complement existing research in evolutionary economics. For example, descriptive approaches on the entry and exit of new enterprises can be assisted by an experiment that can indicate what parameters are responsible for the observed patterns. The model can shed light on why market shares and market size can fluctuate so wildly, and why in the production of one single good there are a variety of technologies being applied. Often such variability has a spatial character as well.

The model here is inherently spatial; agents are located in space, and the transformation of a set of inputs into a composite product can only occur when all products required are present in one location. However, the impact of location depends on the specific model components. To convince the reader that the model is inherently spatial a chapter on transportation costs was added. This topic admittedly is not exhausted — there are a number of issues that still need to be addressed. Some of the issues can actually be solved by applying the full model as developed in this thesis to the case with non-zero transportation costs. An unbalanced economy can become balanced by innovation or even more simply by a change in technology distribution. Both processes have been modelled in this thesis (additional columns to the technology matrices and replacement or copying of skills following the model outlined by Bruckner *et al.* respectively), but due to time and computational constraints these processes were not applied to the simulations involving transportation costs.

Finally, to analyse the artificially generated technologies, a tool was developed to identify any highly connected sets of technologies that might appear in the model output. This tool can be applied to the real world to identify technology clusters as well. In addition, it is possible to detect missing technologies in order to supplement existing unfinished clusters, an application with obvious potential for use in the design of regional technology clusters.

Thus, a lot of ground has been covered here, and there is still a lot of ground to be explored. However, the scope of the phenomena under study is rather large, and the system under study is highly connected. In consequence, it was felt that an integral approach was not only the best, but the only option: none of the components that make up the model could be isolated. Therefore, the result of this thesis is a broad, abstract framework applicable to the study of evolving economic systems. It has the potential to elucidate many aspects of our economy, from dispersal of technology, to location strategies, to pricing, to the development of higher level organisation. Enough is left to do for tomorrow, and who knows what new insights and innovations tomorrow will bring.

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Appendix A

Model parameters

A.1 Parameters for the simulation in Chapter 5

Table A.1: The table provides an overview of the parameters that need to be set at the beginning of the simulation in Chapter 5.

Parameter	Description	Value
ran	seed used to reset the random number generator	50523
rawmaterials	number of raw materials, represented by the first prime numbers	6
lapo	the prime at this position serves as labour	1
mopo	the prime at this position serves as money	2
P	initial product set	{2, 3, 5, 7, 11, 13, 70, 110, 154, 130, 260, 104}
α	rate of return for each of the technologies	2
I and O	initial technology set	See Table 5.1
eqps	equilibrium price set	{1, 1, 2, 2, 2, 0, 10, 10, 10, 2, 2, $\frac{3}{2}$}
maximum productivity	the total level of activity per agent per turn	4
size	the size of the map specified by one side of a square map.	14
wear	proportion of capital that disappears when it is used	$\frac{1}{5}$

Continued on Next Page...

Table A.1 – Continued

Parameter	Description	Value
w	number of iterations indicating the length of the history	50
agents	initial distribution of agents	See Figure 5.1
network	initial set of connections	$\{\}$
buysellconstr	number of products that agent keeps to itself and that are not for sale	maxproductivity
required stock	quantifies an appropriate level of products in store based on the activity levels during history w	
startresources	specifies the quantity of resources a new agent obtains at its introduction	
B_{ij}	birth rate of technology i influenced by technology j through new agent (for this and the following parameters see also Table 5.2)	0
$A_i^{(0)}$	linear birth rate of technology i through new agent	0
$A_i^{(1)}$	second order birth rate of technology i through new agent	0
M_{ij}	parameter controlling the probability of introducing new technology by a new agent is a function of connectivity c and being new n, i.e. probability of introducing new technology i depends on demand for i due to strong technology j	$f(c, n)$
ϕ_i	parameter controlling the spontaneous introduction of technology j by a new agent	0
$A_{ij}^{(1)}$	parameter controlling the probability of replacing technology j by technology i taking into account the size of both fields i and j	$h(gr, (1-gr), o)$
$A_{ij}^{(0)}$	parameter controlling the probability of replacing technology j by technology i taking into account the size of only field j	0
A_{ij}	parameter controlling the probability of innovation by existing agent	$g(gr, c)$

Appendix B

Model algorithms

B.1 The static von Neumann model

Let a and b be the input and output matrices respectively, with dimensions of both matrices $n \times m$, implying that there are m production processes and n products, and let $z(t)$ be the intensity or activity vector at time t. Von Neumann shows that under certain conditions there exist a growth rate α and interest rate β, such that the output at time t, $b \cdot z(t)$ is sufficient to cover the input required at time $t+1$, $a \cdot z(t+1) = a \cdot \alpha z(t)$, and the profitability of each of the production processes is smaller than or equal to the interest rate.

$$a \cdot z(t+1) = a \cdot \alpha z(t) \leq b \cdot z(t)$$

$$\beta \cdot p \cdot a \geq p \cdot b$$

In case there is a single production process for each of the products, the input and output matrices are square, and the activity vector z and price vector p can be found by determining the generalized eigenvectors of a with respect to b. α and β are equal to the largest real eigenvalue (Gale, 1956).

In case there are more production processes than products ($m > n$), the intensity vector z has to be determined differently by using techniques developed for fair matrix games (Kemeny *et al.*, 1956). The set of solutions is given by the solution space of a set of linear inequalities where the extreme points that span the simplex have to be checked for $p \cdot a \cdot z > 0$, i.e. the price and intensity vectors must result in a positive outcome. Any linear combination of the appropriate extreme points is a suitable intensity vector z satisfying the equations above. Furthermore, with a fixed interest rate comes an equilibrium price vector p_{eq} such that each of the production processes has the same return rate.

In any case, if we denote the product set at time t with $S(t) = (S_1, S_2, \ldots, S_n)$ to represent the total quantity of each of the products in the system at time t, then

$$S(t+1) = S(t) + (b - a) \cdot z(t).$$

B.2 The dynamic von Neumann model

Instead of the aggregate activity $z(t)$ per time step now we generate activity bottom up through the use of agents. To explain the different parts of the algorithm in an orderly fashion we make use of a framework provided by Rasmussen and Barrett (1995). Again we define a and b as the input and output matrices with m processes and n products, but now we add r agents to the model. Each of the agent objects A_i is defined by

$$A_i(t) = A_i(f_i, I_{ij}, T_i, S_i(t)) \text{ for } i = 1, \ldots r$$

where f_i gives the activity of agent A_i, I_{ij} is the interaction matrix specifying the agents with whom A_i interacts and T_i specifies which technologies agent A_i possesses. Interactions and activity (which depends on technology T_i) operate on $S_i(t)$, which is the product set in agent A_i's possession at time t. When we assume the agents are in a well stirred vessel, they can interact with all other agents, and $I_{ij} = 1 \forall i, j$.

The activity of the agent A_i is given by

$$f_i(t) = \max_{z_i \in \mathbb{Z}^{m+2n}} (p(t) \cdot (o - i) - \beta) \cdot z_i$$

subject to the conditions **B**, **C**, **T**, **M**, **O**, **F** and **E** listed below,

where $p(t)$ is the vector consisting of the prices $p_i(t)$ for each of the products $i = 1, \ldots, n$ at time t (price mechanism given below). Prices are determined anew for every agent to include the previous agent's effect on the balance of supply and demand. As is standard in this thesis, the first product in the product list is labour and the second product represents one unit of money. Now expand the matrices a and b by joining the matrices a and b with M and I to join the input and output columns of manufacturing, buying and selling:

$$\begin{array}{ccccccccc} & & & manufacturing & & buying & & selling & \\ i & = & (& a & \mid & M & \mid & I &) \\ o & = & (& b & \mid & I & \mid & M &) \end{array}$$

where I is the identity matrix of size n, and M is the square matrix of size n with all elements equal to 0 except for the second row which is equal to $p = \{p_1, p_2, \ldots p_n\}$. z_i is a vector of length $m + 2n$, indicating the actions (manufacturing, buying, selling) of agent i at time t. If we write $z = \{z_1, z_2, \ldots z_{m+2n}\} = \{m_1, \ldots m_m, b_1, \ldots b_n, s_1, \ldots s_n\}$ to break up the activity vector into the different parts of manufacturing (m), buying (b) and selling (s), then agent A_i maximizes the outcome of its actions z_i under the following constraints:

Balance B: The state of agent i at time t minus expenditures plus output has to remain non-negative at all times: $S_i(t) + (o - i) \cdot z_i \geq 0$.

Capital C: An agent cannot execute a technique more often than what it has capital for: $\{z_{c_1}, z_{c_2}, \ldots, z_{c_k}\} \leq \{c_1, c_2, \ldots, c_k\}$ where $c_1, c_2, \ldots, c_k$ are the capital goods required for executing techniques $t_{c_1}, t_{c_2}, \ldots t_{c_k}$.

Technology T: An agent cannot perform actions for which it lacks the skills, and at the same time it cannot perform each individual action more than $maxproductivity$ times: $\{m_1, \ldots, m_m\} \leq maxproductivity \cdot \{t_1, \ldots t_m\}$ where $T_i = \{t_1, \ldots t_m\}$ is the technology list of agent i, which is a boolean vector with 1s and 0s for skills it can and cannot perform.

Maximum activity M: An agent is limited in how much it can do in one turn: $\sum_{i=1}^{m} z_i \leq maxproductivity$.

Offered O: How much an agent can buy depends on the quantities offered:

$\{b_1, \ldots, b_n\} \leq \{\max_{i=1,\ldots,r} S_{i,1}, \max_{i=1,\ldots,r} S_{i,2}, \ldots, \max_{i=1,\ldots,r} S_{i,n}\}$.

For sale F: How much an agent can sell depends on the quantities in possession:

$\{s_1, \ldots, s_n\} \leq S_i(t)$.

Exogenous consumption E: For every consumer the expenditures on consumables $z_{l_1}, z_{l_2}, \ldots z_{l_n}$ have to generate more than c units of labour:

$\{z_{l_1}, z_{l_2}, \ldots z_{l_n}\} \cdot \{o_{l_1}, o_{l_2}, \ldots o_{l_n}\} \geq c$. These consumables are removed from the system without resulting in labour.

With the actions of agent A_i defined as above, its state S_i is updated as follows:

$$S_i(t + \Delta_i) = S_i(t) + (o - i) \cdot z_i(t)$$

The Δ_i indicates that the system is in an intermediate state between $S(t)$ and $S(t+1)$. Once all agents have had their turn, time t is increased to $t + 1$. Thus $\sum_{i=1}^{r} \Delta_i = 1$.

In the execution of $z_i(t)$ the state of other objects is possibly affected as well, since they serve as suppliers or buyers and thus resources have to be exchanged. Appropriate agents (those who have the cash or the produce) are selected randomly and the exchanges are made. This implies that when transaction z_k involves agent $A_{j(k)}$,

$$S_{j(k)}(t + \Delta_i) = S_{j(k)}(t) - (o - i) \cdot \{0, \ldots, 0, z_k, 0, \ldots, 0\} \forall k \in (m+1, \ldots, m+2n)$$

A complete update U_i of the state of the system as a result of the actions of agent A_i thus consist of the following $2n + 1$ instantaneous updates:

$$U_i(S(t)) = \begin{cases} S_i(t+\Delta_i) & = S_i(t) + (o-i) \cdot z_i(t) \\ S_{j(m+1)}(t+\Delta_i) & = S_{j(m+1)}(t) - (o-i) \cdot \{0, \ldots, 0, z_{m+1}, 0, \ldots, 0\} \\ & \vdots \\ S_{j(m+2n)}(t+\Delta_i) & = S_{j(m+2n)}(t) - (o-i) \cdot \{0, \ldots, 0, 0, \ldots, 0, z_{m+2n}\} \end{cases}$$

where $S(t) = (S_1(t), S_2(t), \ldots, S_r(t))$, See Figure B.1. Subsequently, one complete iteration consists of this update sequence applied to each of the agents. Hereto the r agents are put in a randomly ordered list $r_1, r_2, \ldots, r_r$ as Figure B.2 illustrates, and

$$S(t+1) = U_{r_r} \circ U_{r_{r-1}} \circ \ldots \circ U_{r_2} \circ U_{r_1}(S(t))$$

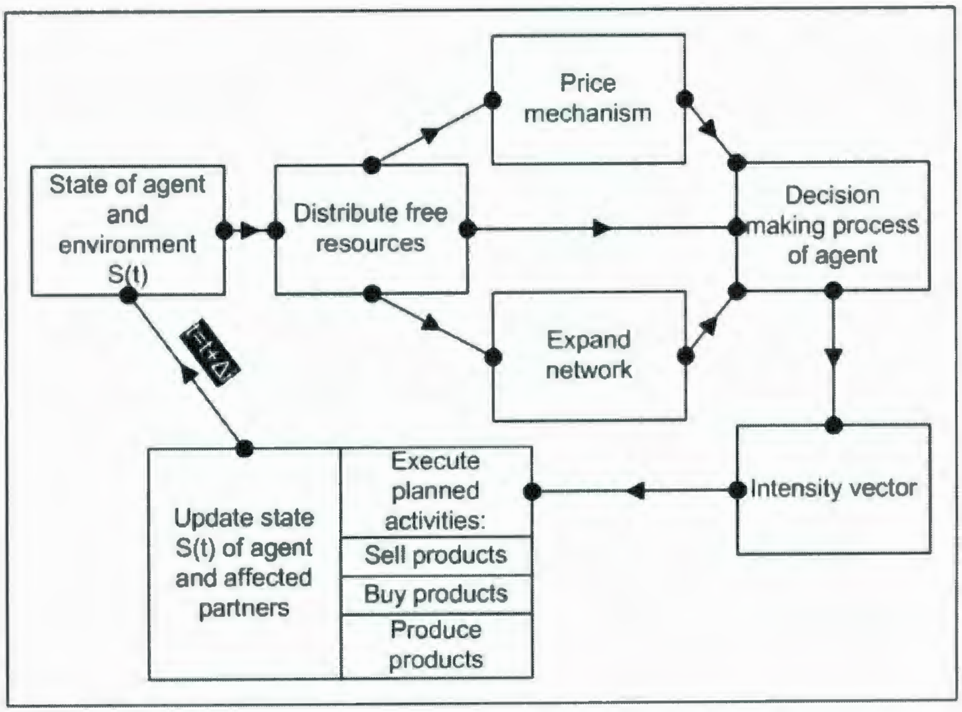

Figure B.1: Activity for one agent cycle. The cycle starts in the top left corner with state $S(t)$ and in particular state $S_i(t)$. After a number of intermediate steps the system is updated to $S(t + \Delta_i)$ and the cycle is repeated but with another agent j.

Furthermore,

$$z(t) = \sum_{i=1}^{r} z_i(t)$$

and

$$S(t+1) = S(t) + (o - i) \cdot z(t).$$

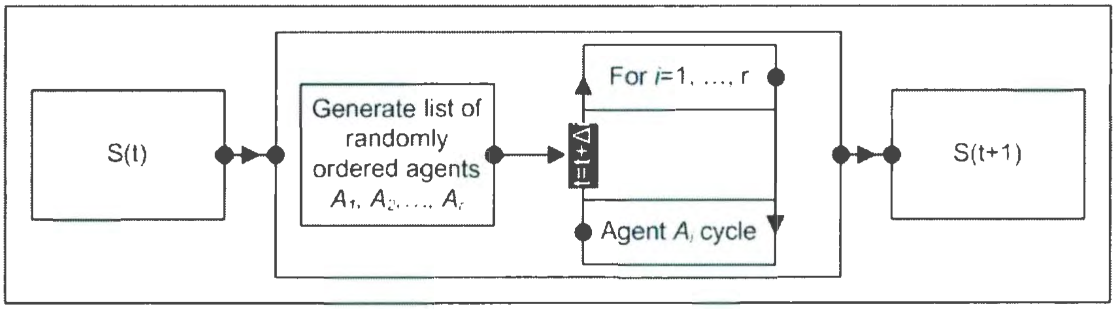

Figure B.2: One iteration cycle. When all agents have had their turn the system has been updated to the next time step.

B.2.1 Price mechanism

The above algorithm defines one iteration. With a history of w iterations it is known which products and how much of each have been demanded during the previous w iterations $D(w, t)$, and also the current stock $\Sigma(t)$ is known. The price mechanism takes into account the balance for each of the n products:

$$B = (bal_1, bal_2, \dots, bal_n) = \Sigma(t) - D(w, t)$$

Von Neumann's work shows there exists a fixed interest/return rate and an equilibrium price p_{eq} that provides each production process with an equal return rate β.

$$\frac{p \cdot (b - a)}{p \cdot a} = (\beta, \dots, \beta)$$

The mechanism to determine appropriate prices $p = \{p_1, \dots, p_n\}$ is a linear programming procedure that minimizes the sum of the prices

$$\min \sum_{i=1,\dots,n} p_i \text{ under the conditions } \mathbf{P} \text{ and } \mathbf{R}$$

where

Price equilibrium P: Prices are always greater than or equal to the equilibrium price: $p \geq p_{eq}$

Return rate R: Let m_p be the set of production processes producing product i for which $bal_i \leq 0$ and let m_l be the set of production processes producing product j for which $bal_j \geq 0$. m_p is the set of manufacturing processes which result in a product of which there currently is a shortage, and m_l is the set of manufacturing processes which result in a product of which there currently is a surplus. In order to balance the market, processes in m_p should produce with a profit, i.e. a return rate higher than the interest rate and processes in m_l should produce at a loss, i.e. a return rate lower than the interest rate. Condition R states that for each $j \in m_p$:

$$\left(\frac{p \cdot (b-a)}{p \cdot a}\right)_j - \beta > 0$$

and for each $j \in m_l$

$$\left(\frac{p \cdot (b-a)}{p \cdot a}\right)_j - \beta \leq 0$$

A solution to the above problem does not always exists. For example there exists no suitable price set when all products are in short supply. To ease this situation prices are set such that condition **R** only applies to the relevant products and production processes, i.e. only to the products involved in the technology of the currently active agent (Remember that prices are set for each agent anew). The currently active agent has no interest in other products, and thus their prices are irrelevant, reducing the number of constraints. It is possible that still no solution exists. In that case the prices are set to the equilibrium price p_{eq} resulting in no opportunity for profit or loss for the active agent, and thus no action. By moving on to the next agent with a different skill set the condition **R** changes and often the deadlock is broken, although this can take a number of agents skipping their turn. When the deadlock is not broken the economy will crash.

B.3 The dynamic spatial von Neumann model

When agents are located on a map instead of in a well stirred vessel they interact with a limited set of agents. This is implemented in the model by a network of connections to indicate which locations are connected with one another. In the structure used above to represent the objects in the model this can be represented by the interaction matrix I_{ij}. Each element i_{ij} is a boolean value, with a 0 for no interaction and a 1 for possible interaction between agent i and j.

Currently when an agent does not have access to all of the products required for its technology, it will attempt to add a connection to its network such that formerly unavailable commodities become available. The implementation of such a process can follow different rules, for example:

- An agent can add the nearest supplier
- An agent can add a supplier at random
- An agent can ask an already existing contact to check his neighbourhood to find a supplier and if one is available this supplier can be added to the network

Whichever process is used, to add agent j to the network of agent i the element j, i of I is updated by replacing a 0 with a 1.

In addition to the limited interaction between agents, the local interaction results in local prices instead of global prices by limiting the balance

$$B_i = (bal_1, bal_2, \ldots, bal_n) = \Sigma(t, i) - D(w, t, i)$$

to the local demand $D(w, i)$ and local current stock $\Sigma(t, i)$ in the network of agent i. No other changes to the model are required.

B.4 The dynamic von Neumann model without exogenous consumption

This is achieved simply by removing condition **E** above. In order to prevent the model from converging to the trivial stable 0-state, f_i is changed:

$$f_i(t) = \max_{z_i \in \mathbb{Q}^{m+2n}} g(t) \cdot z_i \text{ subject to the conditions } \mathbf{B}, \mathbf{C}, \mathbf{T}, \mathbf{M}, \mathbf{O} \text{ and } \mathbf{F}$$

where $g(t) = p(t) \cdot (o - i) - \beta$ with some of its elements replaced. The goal function $g(t)$ provides agents with an indication of the profitability (return rate higher than the interest rate) of each of the production processes. In case $p(t) = p_{eq}$, the return rate for each of the processes is equal to β, and thus none of the processes results in a profit higher than 0. As explained above, the price mechanism sets prices such that the appropriate elements of $g(t)$ are positive, and the remainder of the elements are less than or equal to 0. In order to drive the economy without exogenous consumption, the elements of $g(t)$ corresponding to consumption are replaced by positive numbers, disregarding the actual profits (which are based on the prices and ultimately shortage or surplus of labour) for these elements. This results in a permanent positive stimulus to consume. However, the intensity of the consumption depends on the available means, and thus the rate of consumption is not fixed. Either for a balanced labour supply or one that is unbalanced, consumers are driven to consume. Variations in the size of the inserted positive numbers allows the modelling of consumer preferences.

B.5 Diffusion of technology

Even without new technology and new products being introduced in the model it can be interesting to study the entry and exit of existing technology. As Bruckner *et al.* (1996) showed, successful technology is copied and less successful technology is discarded. This implies that T_i, the technology set in possession of agent i varies with time.

Three indicators are used to define a probabilistic process to simulate diffusion of technology: distance between technology, gross return, and connectivity, and these economic indicators influence the probabilities of changes in the occurrence of technologies in the model.

The similarity between two technologies i and j is based on input coefficients $c_{i1}, \ldots c_{in}$ and $c_{j1}, \ldots c_{jn}$ respectively:

$$d(i,j) = \exp(-\sum_{k=1}^{n} |c_{ik} - c_{jk}|).$$

This results in a measure where for technologies with equal input coefficients the similarity is equal to 1. With increasing differences in coefficients, the similarity will decrease to 0. This measure can be used to define the probability of substituting one technology for another.

Gross return is an appropriate measure to distinguish between the capability of technologies to expand their field. To measure the actual gross return per technology, activity levels per technology of the last w iterations are added and multiplied by their output value to calculate total revenue per technology. Note that this measure is dependent on time t, since production levels will vary over time.

$$gr(t) = (total\ production(t) - total\ production(t-w)) \cdot \frac{\{o(1),\ldots,o(n)\}}{\{N_1,\ldots,N_n\}}$$

Here $o(s)$ is the value of the output of skill s, n is the number of products, and thus, gross revenue $gr(t) = \{gr(1),\ldots gr(n)\}$ is a vector of revenue generated by each of the technologies per agent possessing that technology. Subsequently the vector is normalized by division by the largest element. By normalizing we obtain a measure for success ranging from 0 to 1, with 0 indicating no revenue, and 1 indicating the highest revenue per occurrence of that technology. Moreover, the measure has the same magnitude as the similarity measure defined above (range 0-1), and at the same time $1 - gr$ can be used as a measure for how unsuccessful technologies are; technologies with a score of 1 are the worst performing technologies.

Finally, we define a measure to indicate the connectivity of different technologies. To simulate vertical expansion of an agent, it is interesting to take into account the connections sectors have, either to upstream supplying technologies or to downstream buyers that are being served by the sector (or both). This measure is given by the boolean input-output matrix to indicate the existence of flows between the different sectors.

Subsequently a probability distribution is generated based on the coefficients in Table B.1, and per iteration one of the events in the table is selected (see Figure B.3). After selection the event is applied to a random but appropriate agent.

Table B.1: The definition of entry and exit coefficients. d stands for the similarity matrix, gr is a measure of gross revenue of each of the technologies, and c stands for the connections (between technology) matrix. f and g are functions combining the economic indicators into one matrix. For example, g uses the success score of technology gr with the failure score of technology $(1 - gr)$ to generate a matrix that identifies pairs (f, s) of failing technology f and successful technology s. N_i is the number of occurrences of technology i in the model, i.e. the size of field i.

Type of innovation	Dependency
Innovation by new agent $W_1(N_i+1, N_j \| N_i = 0, N_j) = M_{ij}N_j$	$M_{ij} = c$
Innovation by existing agent $W_2(N_i+1, N_j \| N_i = 0, N_j) = A_{ij}N_j$	$A_{ij} = c$
Spontaneous new agent $W_3(N_i+1) = \phi_i$	$\phi_i = Random(0,1)$
Expansion through new agent $W_4(N_i+1, N_j \| N_i, N_j) =$ $A_i^{(0)}N_i + A_i^{(1)}N_iN_i + B_{ij}N_iN_j$	$B_{ij} = f(gr, d, c),\ A_i^{(0)} = 0,\ A_i^{(1)} = 0$
Imitation $W_5(N_i+1, N_j - 1 \| N_i, N_j) =$ $A_{ij}^{(0)}N_j + A_{ij}^{(1)}N_iN_j$	$A_{ij}^{(1)} = g(gr),\ A_{ij}^{(0)} = 0$

Note that the first two types of innovation in the table deal with new technology, and as such they only apply once the innovation algorithm of the next section is in place.

B.6 Evolving technology

For the implementation of evolving technology the matrices a, b and i, o are no longer fixed in time. In this case the matrices vary with time.

The innovation algorithm pseudo code is given in Figure B.4. The parameters q_1, q_2 and q_3 control the probabilities of innovation and the type of innovation.

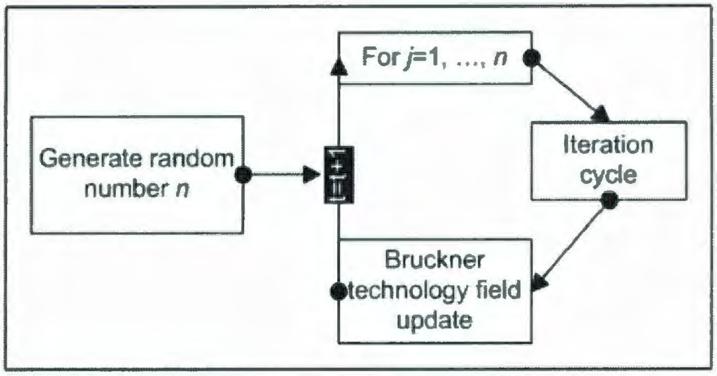

Figure B.3: Probabilistic diffusion of technology. Once every iteration a technology diffusion event is chosen based on the distribution generated by the coefficients in Table B.1.

Suppose that *random* $\leq q_1$ and *random* $> q_2$ such that new technology will be generated. Input for new technology consists of two elements, an actual tool t and the ingredients that are transformed into the desired product by means of tool t. Ingredients are used up in the process, the tool remains available. Once one of the products in short supply is identified, the ingredients for the new technology can be composed. More precisely, when the product in excess demand *ped* is factorized $ped = f_1^{q_1} \cdot f_2^{q_2} \cdot \ldots \cdot f_n^{q_n}$ for some n and $q_. \in \mathbb{N}$, it is clear which factors are required in the ingredients. A random list of products $i_1, \ldots, i_m$ that contain those factors is generated such that the product of these inputs $i = i_1 \cdot i_2 \cdot \ldots \cdot i_m$ will be divisible by the sought after product *ped*, thus $\frac{i}{ped} = g \in \mathbb{N}$. There are then two possibilities: a draw of a uniform random variable is less than q_3 and the new technology assembles the required product *ped*, based on the new input i and tool t, or the uniform random variable is larger than q_3 and the new technology assembles the different parts $i_1, i_2, \ldots, i_m$ into product *ped* without product i first being assembled by means of tool t. g is considered garbage and ignored.

When *random*$\leq q_2$ a new consumable is introduced. Consumers can be modelled such that they do not have a preference for any of the consumables available: they will use whatever is available at the smallest cost. Another option is to let consumer preference correspond to the labour output coefficient. In this case more advanced products can thus be more attractive to the consumer than old fashioned products. A new product is generated by randomly combining existing products. The method to determine the labour output coefficient depends on the type of consumer

```
if random ≤ q1 innovation:
   if random ≤ q2
      then generate new consumable:
            combine random products
            set the quantity of labour resulting from consumption of this good
      else  generate new technology:
            select product that is in excess demand
            create a tool t
            if random ≤ q3
               then new tool requires new product:
                     find list of products with appropriate factorization
                     multiply these products to create new product
               else  new tool t uses old products:
                     find list of products with appropriate factorization
```

Figure B.4: The innovation algorithm in pseudo code. *random* is a uniform random variable in the (0, 1) interval. q_1, q_2 and q_3 are parameters to control the probabilities of innovation and the type of innovation.

preference that is desired.

With the expanding matrices algorithm in place the model can simulate the impact of the introduction of new technology and new products. In order to do so the different processes that have been described in this appendix are placed in a loop depicted in Figure B.5. The agent cycle is repeated again until all agents have had their turn. Subsequently the presence of technology in the model is changed based on a probability distribution that depends on measures for success gr, failure $(1 - gr)$, inter-technology connections c and similarity between technologies d, as was explained in Section B.5. After a random number of repetitions of the iteration-technology diffusion cycle, the matrices are expanded by generating new products and technology, after which the iteration and technology diffusion cycle is repeated again. This process simulates the introduction of new technology and products into a free-market economy, where the success and failure of the products and the technology is determined by the combined action of many local actors.

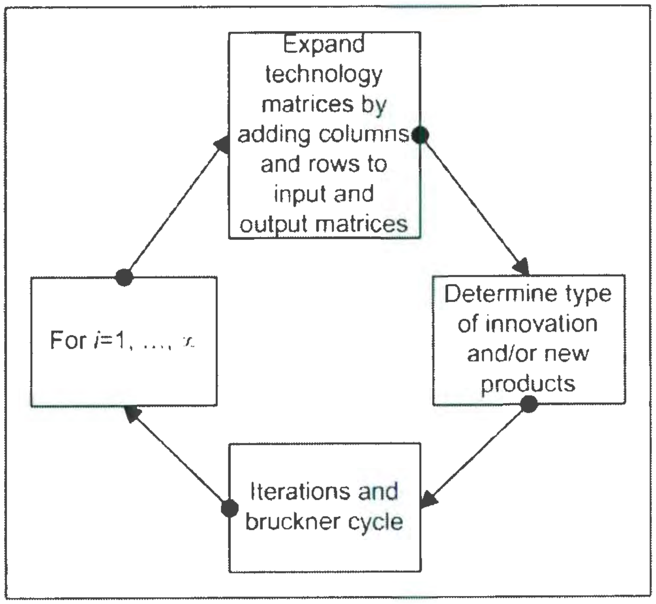

Figure B.5: The full cycle of an evolving economy shows how all of the above cycles are combined into a routine that can be run indefinitely if time would allow.

B.7 Transportation costs

The inclusion of transportation costs involves a different definition of the matrices i and o.

$$\begin{aligned} i &= \left(a \mid M_b \mid I_s \right) \\ o &= \left(b \mid I_b \mid M_s \right) \end{aligned}$$

To implement transportation costs in buying transactions, the matrix M_b, representing the monetary input of all purchases must have a column, not just per product, but per product per trading partner. Trading partners in different locations will have different transportation costs, and this is represented by the enlarged buying input matrix M_b.